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THE IMPACT OF PRIVATE EQUITY ON A LIFE INSURER'S CAPITAL CHARGES UNDER SOLVENCY II AND THE SWISS SOLVENCY TEST

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WORKING PAPERS ON RISK MANAGEMENT AND INSURANCE NO. 91

EDITED BY HATO SCHMEISER

CHAIR FOR RISK MANAGEMENT AND INSURANCE

FEBRUARY 2012

The Impact of Private Equity on a Life Insurer's Capital Charges under Solvency II and the Swiss Solvency Test

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Abstract

In this paper, we conduct an in-depth analysis of the impact of private equity investments on the capital requirements faced by a representative life insurance company under Solvency II as well as the Swiss Solvency Test (SST). Our discussion begins with an empirical performance measurement of the asset class over the period from 2001 to 2010, suggesting that limited partnership private equity funds should be well suited for portfolio diversification, especially from the perspective of life insurers with their large bond holdings. Subsequently, we review the standard approaches for market risk set out by both regulatory regimes and outline a potential framework for an internal model. Based on a concrete implementation of these solvency models for a representative life insurance company, it is possible to demonstrate that private equity is overly penalized by the standard approaches. Hence, life insurers aiming to exploit the asset class's potential may expect significantly lower capital charges when applying an economically sound internal model. Finally, we find that, from a regulatory capital perspective, it can even be less costly to increase the exposure to private rather than public equity.

Keywords: capital requirements, life insurance, portfolio choice, private equity investments, regulation, Solvency II, Swiss Solvency Test

JEL classification: G11; G22; G24; G28; G32

1 Introduction

Within the last decade, the regulation of the European insurance sector was subject to fundamental reforms aimed at the introduction of risk-based solvency standards. One of the first of these initiatives to revise solvency surveillance came into effect in 2004 in the United Kingdom (see, e.g., Cummins and Phillips, 2009). Switzerland followed with its Swiss Solvency Test (SST) in 2006. Beyond these regulatory reforms of individual countries, Solvency II, the EU's flagship project to modernize and harmonize European insurance supervision, has entered its final development phase and is expected to come into force in

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2013. Solvency II and the SST in particular are viewed to be the most innovative frameworks currently available and should thus have a major impact on insurance regulation in the near future. Despite this fact, design and calibration of their standard approaches for market risk clearly promote bond holdings. The prescribed treatment of alternative investments such as private equity, on the contrary, is at least questionable. Since the attractiveness of an asset class for insurance companies does not only depend on its performance characteristics but also on the associated capital charges, an inappropriate assessment from a solvency perspective may cause an underrepresentation of asset classes with favorable risk-return profiles, which would otherwise be well suited for portfolio diversification.

In this paper, we address this issue for the specific case of private equity. Our contribution is twofold. Firstly, identifying its risk-return profile within the critical period from 2001 to 2010, we evaluate the attractiveness of the asset class from a performance perspective and compare it with various investment alternatives. Secondly, we shift our focus to the associated market risk capital requirements for life insurers under Solvency II and the SST. Our discussion begins with a review of the standard approaches for market risk set out by both regulatory regimes as well as an outline of a potential framework for an internal model. Subsequently, we compute and compare the respective capital charges for a stylized life insurance company based on an empirical calibration and implementation of these three solvency models. The results are shown to be robust with regard to the employed benchmark index, the percentage of private equity in the life insurer's portfolio, and the chosen calibration period. Finally, we assess how costly it is, from a regulatory capital perspective, to increase the portfolio weight of private equity in comparison with public equity and hedge funds.

The rest of the paper is organized as follows: Section 2 contains an overview of the literature on private equity and insurance regulation. In Section 3, we briefly discuss the characteristics of private equity investments and conduct an extensive empirical performance analysis, including a comparison with other asset classes. The market risk standard approaches of Solvency II and the SST as well as the outline of an internal model are presented in Section 4. In Section 5, we implement the market risk models and provide an in-depth discussion of the resulting capital charges. Finally, in Section 6 we discuss economic implications and state our conclusion.

2 Literature Review

Since there is a substantial body of extant literature on both private equity and the regulation of financial institutions, we will only review recent research in those areas that are most relevant to our study. We begin with the latest work on the risk-return characteristics as well as the historical performance of the asset class, particularly in comparison with public equity markets. Although industry professionals and general partners regularly stress the attractiveness of private equity investments, existing empirical evidence conveys a rather ambiguous picture. Moskowitz and Vissing-Jorgensen (2002) control for distorted market values and estimate returns for the entire U.S. private equity market, concluding that it did not outperform public equity between 1989 and 1998. Ljungqvist and Richardson (2003),

in contrast, conduct a performance analysis of private equity funds over the two decades from 1981 to 2001 and show that their returns exceeded those of the aggregate public equity market by at least five percent per year. Yet, Zhu et al. (2004) again challenge the allegedly superior risk-return profile of the asset class. They argue that, due to the long investment horizon, severe liquidity constraints, as well as high default probabilities of the portfolio companies, private equity funds hide considerable latent risks, which investors need to take into account. In another empirical paper, Kaserer and Diller (2004) analyze European private equity funds based on cash flow data, documenting an underperformance relative to public equity from 1980 to 2003 but an outperformance during the shorter period from 1989 to 2003. The work of Kaplan and Schoar (2005) is probably one of the most influential studies of private equity performance. Examining the capital flows of more than 1,000 limited partnerships for the period between 1980 and 2001, they demonstrate that the cross-sectional mean return net of fees did virtually not differ from that of the S&P 500. Furthermore, Driessen et al. (2008) devise a statistical methodology based on the generalized method of moments to estimate the risk-return characteristics of non-traded assets from cash flow data. Applying it to a sample of venture capital and buyout funds between 1980 and 2003, they get a mixed impression of the investments' performance. Phalippou and Gottschalg (2009) conduct a performance analysis for an updated version of the Kaplan and Schoar (2005) data set, covering the period from 1980 to 2003. From their results they conclude that the performance figures as disseminated by industry representatives and earlier research are overstated due to sample selection issues and artificially inflated net asset values. When corrected for these biases, the average fund return net of fees falls short of that of the S&P 500 by as much as three percent per year. Finally, Franzoni et al. (2011) provide evidence that private equity returns include a considerable liquidity risk premium. As a consequence, the diversification benefits of the asset class may be lower than traditionally assumed.

Owing to the absence of objective market values, analyses of private equity performance need to rely on appraisal-based figures reported by the limited partnership funds themselves. This, however, causes a variety of problems that are the subject matter of another extensive strand of the private equity literature. Kaplan et al. (2002) aim to assist academics and practitioners with the interpretation of their empirical results by cross-checking the contract specifications of over 140 venture capital financings and pointing out major biases in the two leading private equity databases. Similarly, Woodward (2004) argues that the returns, as disclosed by general partners, are inaccurate measures of the actual changes in value. Introducing an approach that allows to control for this issue, she is able to show that the risk of private equity is higher than commonly assumed, both in terms of return volatility and beta. Another arguably seminal article has been contributed by Cochrane (2005), who applies maximum likelihood estimation to correct the mean returns, volatilities, alphas, and betas of venture capital investments between 1987 and mid-2000 for selection bias. His results indicate that private equity behaves quite similar to traded securities. Conroy and Harris (2007) find understated risks and exaggerated returns based on data for the period from 1989 to 2005, which they attribute to the prevailing practices of appraisal-based portfolio valuation and information disclosure by private equity funds. Systematic valuation biases are also documented by Cumming and Walz (2010), who analyze a data set of more than 5,000 portfolio companies as well as 221 funds between 1971 and 2003. In addition, Cumming et al. (2010a) criticize appraisal-based indices

for being subject to return-smoothing and stale pricing, which results in an underestimation of risk. In order to overcome this issue, they suggest correcting limited partnership private equity benchmarks for autocorrelation. Finally, Korteweg and Sorensen (2010) employ a dynamic sample selection methodology to address the issue of selection bias in venture capital data, and come up with considerably higher risk estimates than earlier research.

Other analyses focus on the effects of the asset class in a portfolio context. Ennis and Sebastian (2005), for example, employ equilibrium pricing to derive an expected return for private equity, which they subsequently use in a mean-variance analysis in order to determine the optimal allocation given different portfolio structures. Schneeweis et al. (2008) consider the diversification benefits of private equity with regard to varying portfolio compositions. Furthermore, Cumming et al. (2010a) conduct a multi-asset portfolio optimization with a modified appraisal-based and a listed private equity index, employing three different risk measures. Their results illustrate that the portfolio allocation decision is quite sensitive to the selected private equity benchmark. Similarly, Cumming et al. (2010b) consider private equity in an optimization framework that incorporates higher central moments of the return distribution and allows them to derive a superior strategic asset allocation compared to the classical Markowitz framework.

With regard to our research objective, we are also interested in previous work on insurance regulation, particularly on Solvency II and the SST. Eling et al. (2007) as well as Steffen (2008) provide overviews of the state of Solvency II at the time of writing of their articles and point out avenues for future research. Moreover, Doff (2008) assesses the Solvency II framework by means of seven conditions that are required for efficient and complete markets. Pfeifer and Strassburger (2008) point out skewness-related stability problems of the Solvency II standard formula and Eling et al. (2008) discuss the main features of the SST as well as its ramifications for the Swiss economy. The implications of Solvency II for the prudential regulation of insurance companies in the U.S. are considered by Vaughan (2009). Cummins and Phillips (2009) additionally conduct an explicit comparison of the U.S. RBC standards, Solvency II, and the SST. Finally, Gisler (2009) compares the insurance risk models embedded in Solvency II and the SST with an emphasis on parameter estimation.

Despite the large number of articles revisited above, research combining the private equity asset class with a regulatory perspective is very scarce. One exception is the work of Cumming and Johan (2007), who draw on a data set of Dutch institutional investors to examine the relationship between regulatory harmonization and portfolio allocations to private equity funds. Besides, Bongaerts and Charlier (2009) analyze regulatory capital requirements for banks' private equity investments under Basel II as well as the Capital Requirements Directive of the European Union, comparing the respective standard approaches with an internal model. The only studies adopting an explicit insurance perspective are provided by Arias et al. (2010) as well as Studer and Wicki (2010). They argue that the LPX50 listed private equity index, which has been proposed by the Committee of European Insurance and Occupational Pension Supervisors (CEIOPS) for the calibration of Solvency II, should be replaced by a more representative index. In their view, the LPX50 is an inappropriate benchmark since it is biased towards leveraged buyout

funds and does not adequately reflect the nature of limited partnership private equity. Consequently, with this paper, we fill a major gap in the literature by investigating the impact of private equity on a life insurer's capital charges under Solvency II and the SST.

3 Private Equity as an Asset Class

In general, "private equity" involves investing in equity securities not listed on a public exchange (see, e.g., Koh, 2009). It provides unlisted firms with long-term share capital in order to nurture their development or restructuring. Investors in this market segment are usually faced with highly illiquid and risky assets and therefore require a compensation by target rates of return that are significantly higher than those for public equity. While there are various reasons for investing in private equity, institutional investors such as banks, insurance companies, or pension funds commonly draw on this asset class in order to diversify their portfolios. Interestingly, recent evidence indicates that a growing fraction of institutional portfolios is invested directly into privately held firms (see, e.g., Dushnitsky, 2012). However, the majority of institutions still prefers an indirect access by becoming a limited partner of a private equity fund (see Nielsen, 2008). The fund itself has a fixed lifetime (usually 10 to 15 years) and is managed by a general partner who calls in money from the limited partners in order to allocate it to target companies (see, e.g., Kaplan and Strömberg, 2009). After the liquidation of an investment, the associated proceeds are distributed to the limited partners.

3.1 Main Investment Styles

Private equity encompasses different investment strategies. According to the age of the firms invested in, the asset class can be subclassified into "venture capital", "growth capital", and "buyouts". Venture capitalists focus on start-up firms that launch and develop the business, whereas growth capital refers to young companies that seek further expansion (see, e.g., Anson, 2002). By contrast, buyouts typically concentrate on the expansion or restructuring of mature companies with stable cash flows (see, e.g., Cendrowski et al., 2008). Since delisting a company often involves large amounts of debt, such deals are also known as "leveraged buyouts" (LBOs). In addition, a transaction can take the form of a management buyout (MBO) aimed at transferring ownership to the company's executives (see Anson, 2002). Other investment styles are summarized by the term "special situations", which comprises the categories of "mezzanine capital" and "distressed financing". Mezzanine capital, a form of subordinated debt, can be viewed as a hybrid between senior secured debt and equity capital. Therefore, it is used when a company is unwilling to issue additional equity but cannot hope for debt financing beyond the level provided by traditional creditors. Furthermore, distressed financing investors (which are often called "vulture investors", see, e.g., Anson, 2002) acquire capital of companies that are in financial distress or close to declaring bankruptcy. The goal of such transactions is to purchase stakes with a great improvement potential at a small fraction of their face values.

To be able to consistently manage their overall portfolios, it is important for investors that fund man-

agers maintain a clear investment style. If a fund deviates from its proclaimed objectives, investors may end up with an unwanted risk-return profile and fall short of their investment targets. This phenomenon is known as style drift and constitutes a particular concern of institutional investors in limited partnership private equity, since the inherent illiquidity of the underlying assets impedes a prompt withdrawal of the invested capital (see, e.g., Cumming et al., 2009).

3.2 Limited Partnerships vs. Listed Private Equity

The term "private equity" only refers to the target investment itself. It does, however, not require that the investing company is privately held. Hence, alongside limited partnership private equity funds, a number of investment vehicles have evolved that target unlisted firms but are themselves publicly listed on a stock exchange. Such constructs are termed "listed private equity" and can adopt three different organizational forms (see Cumming et al., 2010a). First of all, the majority of the listed private equity universe consists of so-called listed direct private equity companies, which directly acquire target firms. Moreover, listed indirect private equity companies invest via limited partnerships and thus are virtually exchange-traded fund of funds. Finally, there is a smaller number of private equity fund managers that invest neither directly nor indirectly but rather set up and manage a portfolio of funds as general partners, thereby generating fee income. While limited partnership private equity funds usually raise most of their capital from large institutions, the investor base of listed private equity tends to be broader. Through listed private equity firms it is easy to gain exposure to a more or less diversified private equity portfolio. Furthermore, the trading in an organized market largely reduces the illiquidity problem of limited partnership funds (see Bergmann et al., 2009).

3.3 Empirical Performance Measurement

3.3.1 Data and Sample Selection

When searching for an adequate index to assess the risk-return characteristics of the private equity asset class, the previously explained distinction between limited partnership funds and listed private equity is quite relevant. Since the advent of listed private equity, there has been a debate among academics and practitioners as to whether or not publicly listed investment vehicles are a useful proxy for the asset class. Some authors claim that listed private equity is strongly influenced by general stock market dynamics and, due to this noisiness, only partly reflects the behavior of the underlying assets (see, e.g., Arias et al., 2010). More specifically, due to the fact that listed private equity is known to trade at discounts to net asset value in times of market turbulence and at premiums to net asset value in growth phases, one might expect an overstatement of the asset class's volatility (see, e.g., Studer and Wicki, 2010). In contrast to these views, proponents of listed private equity state that, apart from the prevalent organizational form, there are hardly any real economic differences to unlisted private equity (see, e.g., Bergmann et al., 2009). As a result, the returns of listed vehicles should be a sufficient proxy for the characteristics exhibited by the asset class. Apart from that, market values arising from regular trading activity are viewed to be more reliable than the rather subjective valuations provided by general partners, which offer the

possibility to smooth returns (see, e.g., Idzorek, 2007).

An attempt to resolve this discussion is beyond the scope of this paper. Instead, we will aim to capture all facets of the asset class by adopting listed as well as limited partnership private equity indices both for the performance measurement and the calibration of the solvency models in Section 4.[1] In this regard, the listed private equity universe will be represented by the LPX50, the LPX Buyout (LPX BO), and the LPX Venture Capital (LPX VC). Each of these benchmarks is published by the Zurich-based LPX Group on a monthly basis and aims to reflect the value development of representative listed private equity funds and firms. We have deliberately chosen the LPX indices over other available listed private equity benchmarks such as the S&P Listed Private Equity Index, the Global Listed Private Equity Index by Red Rocks Capital, or the Privex by Société Générale, since their data series reach further back in time and the main index LPX50 has been employed by CEIOPS for the calibration of the Solvency II standard formula (see CEIOPS, 2010a).[2]

Moreover, the Thomson Reuters Private Equity Performance Index (PEPI), its two subindices for buyout (PEPI BO) and venture capital (PEPI VC), as well as the Cambridge Associates U.S. Private Equity Index (CAPEI) have been selected as benchmarks for limited partnership private equity funds. The PEPI, a common choice for performance measurement purposes among private equity professionals and industry sources such as the U.S. National Venture Capital Association (NVCA), is calculated based on cash flow data and self-reported residual values of over 1,900 limited partnership funds in the Thomson One database. Similarly, the CAPEI incorporates takedowns, distributions, and residual values of 883 limited partnership funds in the U.S., comprising all major investment styles. For all four of these appraisal-based indices so-called pooled end-to-end returns net of fees and carried interest are available on a quarterly basis. Aiming to capture the timing and scale of investments, pooled returns are calculated from a time series of aggregated cash flows, which is derived by summing up the respective figures of all individual limited partnerships that enter the index. Thus, in a sense, all constituents are treated as if they were one single fund. Resulting in investment-weighted instead of average returns, this methodology is supposed to more closely mirror the performance characteristics of typical private equity portfolios.[3]

For comparison purposes we also consider a range of common benchmarks, representing other asset classes. MSCI country indices are employed as measures for the U.S. (MSCI USA), European (MSCI EU), and Swiss (MSCI CH) stock markets. Note that these have been selected because they are also among the major stock market risk factors within the SST standard model (see Section 5). In addition, the development of the corresponding government bond markets will be captured through the S&P U.S. Treasury Index (S&P USTI), the S&P Eurozone Government Bond Index (S&P EUGI), as well as the

[1] In fact, it is not uncommon that institutional investors hold listed and limited partnership private equity at the same time. The decision of how much capital is allocated to each market segment thereby depends on a variety of firm specific factors (see Cumming et al., 2011).

[2] Each of the LPX indices dates at least back to January 1998, while none of the other listed private equity benchmarks has a history before January 2004. See www.lpx.ch, www.standardandpoors.com, www.redrockscapital.com, and www.sgindex.com for more information.

[3] A corollary of this method is that larger funds exert more influence on the performance than smaller funds. Refer to the documentation of the Thomson One database for additional details.

Swiss Government Bond Index (SIX SBI). Finally, the HFRX Global Hedge Fund Index (HFRX) and the HFRI Fund Weighted Composite Index (HFRI) have been chosen as proxies for hedge fund investments. Data for all of these benchmarks has been obtained from Bloomberg. Wherever available, total return indices are used to account for coupons and dividends, which would otherwise not be reflected in prices.

The time horizon for a performance analysis is frequently determined arbitrarily or through restrictions on the available data. Here, we decided to opt for the decade from January 2001 to December 2010, since this period has been chosen by FINMA for the calibration of the 2011 SST standard model for market risk. Thus, monthly and quarterly log-return time series for that period have been collected for each previously mentioned index so as to ensure consistency with the empirical results in Section 4. Furthermore, the average one-month (log) T-Bill rate p.a. (2.02%) and the average three-month (log) T-Bill rate p.a. (2.17%) between January 2001 and December 2010 have been used as risk-free interest rates for the calculation of performance measures based on monthly and quarterly index returns, respectively.

3.3.2 Private Equity in Comparison with Other Asset Classes

With the relevant data at hand, we can now analyze the risk-return profile of private equity in comparison with that of other common asset classes. Table 1 summarizes the empirical results. In order to preserve the typical characteristics of each asset class, we deliberately refrained from converting the time series of the indices denominated in euros and Swiss francs into U.S. dollars, thus implicitly assuming that the investor is able to sufficiently hedge exchange rate fluctuations at an immaterial cost.[4] Since hedging foreign investments against exchange rate risk is very common among institutional investors and the trading costs for the required foreign exchange (FX) instruments such as futures and options are relatively small,[5] we believe this to be an acceptable assumption for our purpose.

When comparing the descriptive statistics in the upper parts of Panel A and B in Table 1, we notice that the PEPI BO (9.80 percent p.a.), the CAPEI (9.60 percent p.a.), and the PEPI (6.97 percent p.a.) exhibited the highest mean returns of all indices under consideration. Only the broad hedge fund index HFRI (6.70 percent p.a.) comes relatively close to these figures, followed by the government bond benchmarks S&P USTI (4.91 percent p.a.), S&P EUGI (4.76 percent p.a.), and SIX SBI (4.20 percent p.a.). And while the HFRX (3.50 percent p.a.) still delivered a positive excess return, stocks, listed private equity, as well as the PEPI VC generated mean returns below the risk-free interest rate. Furthermore, considering the annualized standard deviation of returns of the LPX50 (26.70 percent p.a.), the LPX BO (27.01 percent p.a.), and the LPX VC (27.92 percent p.a.), we find that listed private equity has been by far the most volatile investment, followed by the MSCI EU (20.29 percent p.a.), the MSCI USA (16.73 percent p.a.), and the MSCI CH (15.24 percent p.a.). Consistent with this observation, the maximum and minimum returns are also furthest apart for the three listed private equity and the three stock indices. In contrast to that, the volatilities of the limited partnership private equity benchmarks lie between 11.61

[4] Consider, for example, an investment in a foreign government bond. If left completely unhedged, this would be an outright speculation on the exchange rate, as the returns in the investor's home currency will be dominated by currency fluctuations, implying that the asset does not exhibit the typical behavior of a government bond.

[5] Flat fees for FX futures traded at the Chicago Mercantile Exchange (CME), the largest regulated FX marketplace worldwide, can be as low as USD 0.11, depending on membership and volume. For more information see www.cmegroup.com.

percent p.a. (CAPEI) and 12.48 percent p.a. (PEPI VC), thus being less than half as high as those of their listed counterparts. Besides, the HFRI (6.50 percent p.a.) and the HFRX (5.96 percent p.a.) mark the beginning of the lower end of the range and, according to the S&P USTI (4.29 percent p.a.), the S&P EUGI (3.63 percent p.a.), as well as the SIX SBI (3.83 percent p.a.), government bonds were not only the least volatile investment but also generated the highest minimum returns over our examination horizon. In terms of negative months, however, the private equity indices CAPEI (30) and PEPI BO (33) did even better than the three government bond indices.

In addition to these descriptive statistics we have calculated the two widespread performance measures Sharpe Ratio (see Sharpe, 1966) and Calmar Ratio (see Young, 1991) and established the corresponding rank orders for the sample under consideration. The results for each index are displayed in the lower parts of Panels A and B in Table 1. At first glance, we notice that the three listed private equity indices, the three stock indices, as well as the PEPI VC are not ranked. This is due to the fact that their mean returns fell short of the risk-free interest rate. The Sharpe and Calmar Ratio, however, are not meaningful for negative excess returns since, in that case, a higher value of the risk measure in the denominator would be associated with a better performance (i.e. a less negative ratio). The top three among those indices with positive Sharpe Ratios are the S&P EUGI (0.7529), the HFRI (0.7196), as well as the S&P USTI (0.6732), closely followed by the CAPEI (0.6406) and the PEPI BO (0.6219) on ranks four and five. Thus, two of the four appraisal-based private equity indices end up in the first third of the total sample, outperforming the SIX SBI (0.5686), the PEPI (0.4096), as well as the HFRX (0.2472) on ranks six, seven, and eight.[6] Turning to the Calmar Ratio we notice that, although the HFRX (0.0125) is ranked eighth again, the order for the other indices changes considerably. More specifically, the best performance in the sample is now attributed to the PEPI BO (0.1158) and the CAPEI (0.1111). In addition, the PEPI (0.0747) rises up to rank five, just behind the two government bond benchmarks SIX SBI (0.0985) and S&P EUGI (0.0819). Finally, the S&P USTI (0.0675) as well as the HFRI (0.0550) rank sixth and seventh.

To complete the analysis, we look at the correlation structure between private equity and the other indices in our sample, which is shown in Table 2. Based on a common t-test we find all Bravais-Pearson correlation coefficients to be significantly different from zero. Interestingly, private equity exhibited unexpectedly high positive correlations of between 0.68 and 0.82 with the three stock indices, regardless of whether we consider limited partnerships or listed vehicles. Thus, it appears as if, during our examination period, the asset class was not quite able to decouple from the developments in the public equity markets. Furthermore, we find positive correlations with the hedge fund indices of between 0.39 and 0.79. The correlation coefficients between private equity and government bond returns, on the contrary, turned out to be moderately negative, ranging between –0.52 and –0.20. Consequently, private equity seems to offer excellent diversification qualities for investors with large bond portfolios such as life insurers.

[6] One reason for the superior mean-variance profile of the HFRI compared to the HFRX and the resulting outperformance based on the Sharpe Ratio is likely to be its higher degree of diversification. With more than 2,000 constituents, the HFRI is a considerably broader index than the HFRX. See www.hedgefundresearch.com for more information.

Panel A: Private Equity	**LPX50**	**LPX BO**	**LPX VC**	**PEPI**	**PEPI BO**	**PEPI VC**	**CAPEI**
Bloomberg Ticker	LPX50TR	LPXABOTR	LPXVENTR	-	-	-	-
Currency	EUR	EUR	EUR	USD	USD	USD	USD
Return Interval	Monthly	Monthly	Monthly	Quarterly	Quarterly	Quarterly	Quarterly
Total Return over Period	–21.95%	19.92%	–86.43%	69.73%	97.96%	–18.49%	96.02%
Mean Return	–0.18%	0.17%	–0.72%	1.74%	2.45%	–0.46%	2.40%
– *Annualized* –	–2.19%	1.99%	–8.64%	6.97%	9.80%	–1.85%	9.60%
Standard Deviation	7.71%	7.80%	8.06%	5.87%	6.13%	6.24%	5.80%
– *Annualized* –	26.70%	27.01%	27.92%	11.73%	12.27%	12.48%	11.61%
Maximum	28.53%	30.54%	25.30%	10.63%	12.18%	7.93%	14.20%
– *Annualized* –	342.31%	366.50%	303.58%	42.50%	48.71%	31.71%	56.81%
Minimum	–30.66%	–35.82%	–23.11%	–16.08%	–16.46%	–16.64%	–16.74%
– *Annualized* –	–367.97%	–429.79%	–277.28%	–64.30%	–65.86%	–66.56%	–66.94%
Number of Negative Months	46	41	61	45	33	54	30
Sharpe Ratio	–0.1580	–0.0012	–0.3821	0.4096	0.6219	–0.3217	0.6406
– *Rank* –	–	–	–	7	5	–	4
Calmar Ratio	–0.0115	–0.0001	–0.0385	0.0747	0.1158	–0.0603	0.1111
– *Rank* –	–	–	–	5	1	–	2

Panel B: Other Asset Classes	**MSCI USA**	**MSCI EU**	**MSCI CH**	**S&P USTI**	**S&P EUGI**	**SIX SBI**	**HFRX**	**HFRI**
Bloomberg Ticker	GDDLUS	GDDLEMU	GDDLSZ	SPBDUSBT	SPBDEGIT	SBIDGT	HFRXGL	HFRIFWI
Currency	USD	EUR	CHF	USD	EUR	CHF	USD	USD
Return Interval	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly	Monthly
Total Return over Period	14.78%	–7.71%	1.78%	49.11%	47.59%	42.00%	34.99%	66.98%
Mean Return	0.12%	–0.06%	0.01%	0.41%	0.40%	0.35%	0.29%	0.56%
– *annualized* –	1.48%	–0.77%	0.18%	4.91%	4.76%	4.20%	3.50%	6.70%
Standard Deviation	4.83%	5.86%	4.40%	1.24%	1.05%	1.10%	1.72%	1.88%
– *Annualized* –	16.73%	20.29%	15.24%	4.29%	3.63%	3.83%	5.96%	6.50%
Maximum	9.17%	14.99%	11.52%	4.41%	3.70%	4.49%	3.10%	5.02%
– *Annualized* –	110.02%	179.92%	138.18%	52.95%	44.40%	53.83%	37.21%	60.22%
Minimum	–18.76%	–19.27%	–13.24%	–3.56%	–2.78%	–1.84%	–9.81%	–7.09%
– *Annualized* –	–225.07%	–231.25%	–158.82%	–42.74%	–33.41%	–22.10%	–117.76%	–85.05%
Number of Negative Months	46	53	51	39	42	43	39	37
Sharpe Ratio	–0.0326	–0.1378	–0.1211	0.6732	0.7529	0.5686	0.2472	0.7196
– *Rank* –	–	–	–	3	1	6	8	2
Calmar Ratio	–0.0024	–0.0121	–0.0116	0.0675	0.0819	0.0985	0.0125	0.0550
– *Rank* –	–	–	–	6	4	3	8	7

Table 1: Descriptive Statistics and Performance Measures (01/01/2001 – 12/31/2010)

Panel A: Descriptive statistics and performance measures for the log-return time series of seven widely recognized private equity indices. Listed private equity: LPX50; LPX Buyout (LPX BO); LPX Venture Capital (LPX VC). Limited partnership private equity funds: Thomson Reuters Private Equity Performance Index (PEPI), including the subindices for buyout (PEPI BO) and venture capital (PEPI VC); Cambridge Associates U.S. Private Equity Index (CAPEI). **Panel B**: Descriptive statistics and performance measures for the log-return time series of eight common indices, representing established asset classes. Stock markets: MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH). Government bond markets: S&P U.S. Treasury Index (S&P USTI), S&P Eurozone Government Bond Index (S&P EUGI), and Swiss Government Bond Index (SIX SBI). Hedge Funds: HFRX Global Hedge Fund Index (HFRX) and HFRI Fund Weighted Composite Index (HFRI). Risk-free rate: average one-month (log) T-Bill rate p.a. (2.02%) for indices with monthly return interval, and average three-month (log) T-Bill rate p.a. (2.17%) for indices with quarterly return interval. Note: for the time series with quarterly frequency, the number of negative months has been calculated by simply tripling the number of negative quarters, thus neglecting the possibility of a negative quarter comprising one or two positive months.

	MSCI USA		MSCI EU		MSCI CH		S&P USTI		S&P EUGI		SIX SBI		HFRX		HFRI	
LPX50	0.82	***	0.82	***	0.79	***	–0.38	***	–0.26	***	–0.27	***	0.60	***	0.77	***
LPX BO	0.76	***	0.75	***	0.72	***	–0.31	***	–0.22	**	–0.24	**	0.57	***	0.69	***
LPX VC	0.74	***	0.77	***	0.75	***	–0.35	***	–0.20	*	–0.21	**	0.50	***	0.72	***
PEPI	0.81	***	0.78	***	0.76	***	–0.50	***	–0.46	***	–0.47	***	0.63	***	0.78	***
PEPI BO	0.78	***	0.74	***	0.72	***	–0.46	***	–0.42	**	–0.44	***	0.64	***	0.75	***
PEPI VC	0.70	***	0.68	***	0.69	***	–0.42	**	–0.44	***	–0.42	**	0.39	**	0.64	***
CAPEI	0.80	***	0.77	***	0.75	***	–0.52	***	–0.48	***	–0.44	***	0.65	***	0.79	***

Table 2: Correlations between Private Equity and Other Asset Classes (01/01/2001 – 12/31/2010)

Significance levels of correlation t-test: ***=1%; **=5%; *=10%. Listed private equity (correlations based on monthly returns): LPX50; LPX Buyout (LPX BO); LPX Venture Capital (LPX VC). Limited partnership private equity funds (correlations based on quarterly returns): Thomson Reuters Private Equity Performance Index (PEPI), including the subindices for buyout (PEPI BO) and venture capital (PEPI VC); Cambridge Associates U.S. Private Equity Index (CAPEI). Other indices: U.S. equities (MSCI USA); European equities (MSCI EU); Swiss equities (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); HFRX Global Hedge Fund Index (HFRX); HFRI Fund Weighted Composite Index (HFRI).

Clearly, our results need to be interpreted in light of the chosen examination period from January 2001 to December 2010, which has been shaped by several years of major market dislocations due to the burst of the dot-com bubble in 2001 and the global financial crisis of 2007 to 2009. Thus, this is a relatively unfavorable decade for risky asset classes, while it certainly bolsters the observed performance of government bonds, which are heavily sought after in times of market turbulence.[7] Nevertheless, we observe an outperformance of private equity relative to public equity and, depending on the applied performance measure and benchmark index, even hedge funds. Against this background, it is safe to say that, with the exception of the remarkable weakness in the venture capital segment, we get a solid impression of the asset class. As mentioned in Section 2, however, one also needs to bear in mind that an analysis based on self-reported values might, at least to some extent, be subject to distortions. In particular, the fact that listed private equity has done very poorly during the same decade could be an indication for the presence of return smoothing or a potential sample selection bias in our results for the appraisal-based indices. Yet, at the same time these huge differences in the observed performance raise doubts with regard to the suitability of publicly listed vehicles as a measure for the behavior of the actual underlying assets. In any case, our results illustrate that, within reasonable limits, private equity should be well suited for portfolio diversification. As already mentioned, however, the historical performance of an asset class is not the only key factor when deciding about a potential investment. Instead, financial institutions also need to consider the associated capital charges. Hence, in the remainder of this paper, we will conduct an in-depth analysis of the treatment of private equity under Solvency II and the SST.

[7] Unreported results for the period between January 1998 and December 2006 indicate a considerably better performance of both limited partnership funds and listed private equity.

4 Solvency Assessment and Market Risk

4.1 Solvency II Market Risk Module

Under Solvency II, the solvency capital requirement (SCR) for insurance companies can be calculated based on a standard approach that has been calibrated by the regulator so as to correspond to a value at risk approach with a confidence level of 99.5 percent and a one-year time horizon (see EC, 2010; CEIOPS, 2010a).[8] In this paper, we focus on the market risk module. A key concept in this regard is the so-called net asset value (NAV), which equals the assets less the liabilities when both magnitudes are valued according to the prices achievable in an arm's-length transaction (see EC, 2010). To calculate the overall SCR for market risk, the changes in the net asset value (ΔNAV) caused by preset shocks to various capital market variables need to be aggregated (see CEIOPS, 2009). For reasons of simplicity and comparability, we restrict ourselves to the capital requirements for interest rate fluctuations (Mkt_{int}), stock market movements (Mkt_{eq}), and shifts in real estate prices (Mkt_{prop}).

According to EC (2010), interest rate risk is defined as the change in the net asset value (ΔNAV) that is caused by movements of the term structure of interest rates (see, e.g., EC, 2010). Since, in general, assets and liabilities of insurance companies are interest rate sensitive, both upward and downward shocks to the yield curve have an effect on NAV. Thus, Mkt_{int} is distinguished in two situations:

$$Mkt_{int}^{up} = \Delta NAV|_{up}, \tag{1}$$

$$Mkt_{int}^{down} = \Delta NAV|_{down}. \tag{2}$$

Mkt_{int}^{up} and Mkt_{int}^{down} capture ΔNAV due to a preset upward and downward shock, respectively. The corresponding stresses for each interest rate r_t in the term structure are applied as follows (see EC, 2010):

$$\begin{aligned} &r_t \cdot (1 + s_t^{up}) \quad \forall t, \text{ for the upward shock,} \\ &r_t \cdot (1 + s_t^{down}) \quad \forall t, \text{ for the downward shock,} \end{aligned} \tag{3}$$

where t stands for the maturity and s_t^{up} as well as s_t^{down} equal the shocks for the up and down state.

Mkt_{eq} is based on the change in net asset value due to a drop in equity prices. In the Solvency II proposal, the equity risk category is split into "global equity" and "other equity" (see EC, 2010). The former contains equity investments that are listed on an organized market of an OECD or EEA country, whereas the latter comprises non-listed equities, emerging market stocks, commodities, hedge funds, and any other investments that are not considered in one of the remaining risk categories. The calculation of Mkt_{eq} is carried out in two steps (see EC, 2010). First of all, the capital requirements for each of the

[8] We concentrate on the SCR since it is the key measure of Solvency II. Another magnitude is the so-called "minimum capital requirement (MCR)", which is governed by a linear formula including a certain percentage of the solvency capital requirement as floor and cap (for further information refer, e.g., to EC, 2010).

two equity subcategories are determined by the impact of a prespecified shock:

$$Mkt_{eq,i} = \max\left(\Delta NAV \mid \text{equity shock}_i, 0\right), \tag{4}$$

with $i = \{\text{global equities; other equities}\}$. In a second step, Mkt_{eq} is calculated based on a given correlation structure between global and other equities (see EC, 2010):

$$Mkt_{eq} = \sqrt{Mkt_{eq,i} \cdot Mkt_{eq,j} \cdot Corr_{eq}}, \tag{5}$$

where $i, j \in \{\text{global equities; other equities}\}$ and $Corr_{eq}$ denotes the applicable correlation coefficient.

Similar to the capital requirement for equity risk, Mkt_{prop} is calculated on a prespecified loss in real estate values (see EC, 2010):

$$Mkt_{prop} = \max\left(\Delta NAV \mid \text{property shock, } 0\right). \tag{6}$$

Finally, the overall capital charges for market risk, SCR_{Mkt}, can be determined as (see EC, 2010):

$$SCR_{Mkt} = \max\left\{\sqrt{\sum Corr^{up}_{Mkt} \cdot Mkt^{up}_i \cdot Mkt^{up}_j}; \sqrt{\sum Corr^{down}_{Mkt} \cdot Mkt^{down}_i \cdot Mkt^{down}_j}\right\}, \tag{7}$$

where $i, j \in \{int; eq; prop\}$, the superscripts denote the up and down state for interest rate risk, and $Corr^{up}_{Mkt}$ as well as $Corr^{down}_{Mkt}$ are the preset correlation coefficients, which can be found in Table 7 of the Appendix. Mkt_{eq} and Mkt_{prop}, are independent of the interest rate scenarios.

4.2 SST Standard Model for Market Risk

In this section, we discuss the main features of the SST standard model for market risk as laid down by the Swiss Federal Office of Private Insurance (FOPI) and the Swiss Financial Market Supervisory Authority (FINMA). The SST is based on the market-consistent valuation of an insurance company's assets and liabilities (see FOPI, 2004). A key magnitude for the solvency assessment is the so-called risk-bearing capital at time t (RBC_t), which is defined as the market value of the assets (A_t) minus the best estimate of the liabilities (L^b_t). It represents the firm's available reserve to cope with fluctuations in assets and liabilities over time (see FOPI, 2006):

$$RBC_t = A_t - L^b_t. \tag{8}$$

Since there are no liquid markets for insurance liabilities, the market-consistent value of the liabilities (L_t) in the context of the SST is derived by adding a model-based market value margin (MVM_t) to the best estimate value of the liabilities (see FOPI, 2006):[9]

$$L_t = L^b_t + MVM_t. \tag{9}$$

[9]For a discussion of valid approaches to the market-consistent valuation of insurance liabilities, refer to FOPI (2004).

This implies that the market value of the assets is higher than the market value of the liabilities if the risk-bearing capital exceeds the market value margin:

$$\begin{aligned} RBC_t &= A_t - (L_t - MVM_t) \\ RBC_t - MVM_t &= A_t - L_t. \end{aligned} \tag{10}$$

The market value margin represents the hypothetical capital that would be required for an orderly runoff of the liabilities in case of an insolvency of the insurer, either by closing for new business and exhausting the remaining capital or by transferring the liabilities of the insolvent firm to another institution.[10] In contrast to Solvency II, the SST adopts the tail value at risk (also called conditional value at risk or expected shortfall) as risk measure. The value at risk for the confidence level $1-\alpha$ (e.g., 99.5%), VaR_α, is generally defined as the loss over a particular period that is only exceeded with probability α. If the random variable X under consideration represents value changes (e.g., returns), the VaR equals the α-quantile (Q_α) of the respective distribution. In line with this definition, the tail value at risk, TVaR_α, is the conditional expected value for those realizations of X that are equal to or lower than the VaR_α: $\mathrm{TVaR}_\alpha = \mathrm{E}[\mathrm{X}|\mathrm{X} \leq \mathrm{VaR}_\alpha]$. Hence, the TVaR_α represents the size of the average loss in case the VaR_α is exceeded. For the SST, FINMA has set the confidence level to 99 percent, implying an exceedance probability α of one percent (see FOPI, 2006).

Moreover, in line with common practice in the financial services industry, the SST is based on a risk evaluation period of one year. The current values of the assets and liabilities determine their possible realizations at the end of the year and, in turn, the remaining risk-bearing capital. Therefore, the insolvency probability of an insurance company can be controlled by providing for an appropriate level of risk-bearing capital in $t = 0$. Against this background, the SST target capital (TC) is defined as the amount of risk-bearing capital today (RBC_0) for which the conditional expected value of the one percent lowest levels of risk-bearing capital at the end of the year ($R\tilde{B}C_1$) equals the market value margin MVM_1 (see FOPI, 2006):

$$E[R\tilde{B}C_{1|RBC_0=TC} \mid R\tilde{B}C_{1|RBC_0=TC} \leq Q_{1\%}] = MVM_1. \tag{11}$$

To put it differently, the regulator prescribes an amount of RBC_0 which ensures that, even for the most detrimental outcomes, $R\tilde{B}C_1$ is on average sufficient to cover the costs of running off the insurance portfolio, i.e. MVM_1. It can be shown that the following formulation is equivalent to this definition of TC (see FOPI, 2006):

$$\begin{aligned} TC &= -\mathrm{TVaR}_{1\%}\left(\frac{R\tilde{B}C_1}{(1+r_f)} - RBC_0\right) + \frac{MVM_1}{(1+r_f)} \\ &= -\mathrm{TVaR}_{1\%}\left(\Delta R\tilde{B}C\right) + \frac{MVM_1}{(1+r_f)}, \end{aligned} \tag{12}$$

[10] If the market value margin was unavailable as a compensation for the necessary regulatory capital cost, potential third-party investors would not be willing to acquire the portfolio of insurance policies.

where r_f denotes the risk-free interest rate and $\Delta\tilde{RBC}$ is the change in risk-bearing capital.

Thus, the target capital (TC) in $t = 0$ is defined as the sum of the (discounted) expected value of the one percent largest declines in risk-bearing capital and the present value of the market value margin. In order to compute TC, the probability distributions for the random variables $\tilde{RBC}_1$ and $\Delta\tilde{RBC}$ are required. For this purpose, the SST includes a standard model, which serves to describe the stochasticity of the change in risk-bearing capital due to changes in a wide range of market risk factors such as stock prices, interest rates, credit spreads, real estate prices, and exchange rates.[11] The k-dimensional random vector of the changes in the risk factors, $\Delta\tilde{RF} = (\Delta\tilde{f}_1, ..., \Delta\tilde{f}_k)'$, is assumed to be multivariate normally distributed with individual means of zero and the variance-covariance matrix Σ (see FOPI, 2004):

$$\Delta\tilde{RF} \sim N_k(\mathbf{0}, \Sigma). \tag{13}$$

Due to the assumption of multivariate normally distributed risk factor changes, the distribution of $\Delta\tilde{RBC}$ is also normal and can be derived based on the mean and variance of $\Delta\tilde{RF}$ (see Appendix A).

However, for some risk factor changes the normality assumption is a more or less strong simplification of reality.[12] In order to mitigate this issue, the analytical model for $\Delta\tilde{RBC}$ is complemented by a number of historical and hypothetical capital market stress scenarios, which account for deviations from normality. The goal is to generate a more sophisticated representation of the highly relevant tail characteristics of $\Delta\tilde{RBC}$. FOPI (2006) describes the scenarios as events that occur with a low frequency but have a severe impact on the risk-bearing capital of an insurance company. Suppose that only one of m possible scenarios can occur once in any year and consider the following distinction of cases:

- S_0 : base case (normal year without a scenario),
- S_j : occurrence of scenario j $\quad \forall j \in \{1, ..., m\}$.

The normal years and the specific scenarios occur with probabilities p_0 and p_j, respectively, and are assumed to be mutually exclusive, implying:

$$p_0 = 1 - (p_1 + p_2 + ... + p_m).$$

For the 2011 SST, a range of $m = 11$ scenarios is preset by the regulator. Apart from these, insurance companies are free to add custom scenarios that are of particular importance to their business situation. In general, scenarios consist of stresses with regard to several risk factors, which can be translated into a deterministic total change in risk-bearing capital, c_j, caused by the occurrence of the respective scenario:

$$c_j = \Delta RBC(S_j) \quad \forall j \in \{1, ..., m\}. \tag{14}$$

Subsequently, this c_j is used to derive a cumulative distribution function (cdf) prevailing under each

[11] A comprehensive list of the current 79 risk factors can be found in FINMA (2010).
[12] Stock returns, for example, have been repeatedly shown to exhibit skewness and excess kurtosis (see, e.g., Officer, 1972).

scenario. To this end, consider the cdf for $\Delta R\tilde{B}C$ in the base case:

$$F_0(x) = P\left(\Delta R\tilde{B}C \leq x|S_0\right). \tag{15}$$

It is now assumed that under each scenario, all possible changes in the risk-bearing capital are lowered by an amount of c_j compared to a normal year. Through this assumption potential scenario-induced deformations of the distribution, such as changes in skewness and kurtosis, are ignored (see FOPI, 2006). Hence, the cdf for scenario S_j is obtained by shifting the cdf of the base case accordingly:

$$F_j(x) = P\left(\Delta R\tilde{B}C \leq x|S_j\right) = F_0(x - c_j) \quad \forall j \in \{1, ..., m\}. \tag{16}$$

Finally, the scenarios and the normal year are consolidated into an aggregate cdf for $\Delta R\tilde{B}C$, which equals the weighted mean of the individual probability distributions(see FOPI, 2006):

$$F(x) = \sum_{j=0}^{m} p_j F_j(x) = \sum_{j=0}^{m} p_j F_0(x - c_j). \tag{17}$$

Based on this mixture of $m + 1$ normal distributions, $F(x)$, it is possible to determine the target capital according to Equation (12).

4.3 Outline of an Internal Model for Market Risk

Apart from the standard approaches under Solvency II and the SST, insurance companies can also rely on internal models to calculate their capital charges for market risk. Hence, in this section we introduce such an alternative framework that will serve us to draw the desired comparisons in the context of the empirical analysis in Section 5. In essence, our approach is a parsimonious structural credit model in which default is characterized as the company's asset value being insufficient to repay its liabilities.[13] Instead of employing risk factor fluctuations, which are then translated into changes in the firm's risk-bearing capital, our model directly builds upon the stochasticity of the market values of asset classes and liability categories. Consider a one-period evaluation horizon and continuous compounding. In addition, assume that the life insurer has a stable client base and cash flows are exchanged at the beginning of the period. Under this setup, the assets in $t = 1$ can be described as follows:

$$\tilde{A}_1 = A_0 \exp(\tilde{r}_A), \tag{18}$$

where

- $\tilde{A}_1$: stochastic market value of the assets in $t = 1$,
- A_0: deterministic market value of the assets in $t = 0$,
- $\tilde{r}_A$: stochastic return on the assets between $t = 0$ and $t = 1$.

[13] Structural credit models, which can be traced back to the seminal work of Merton (1974), are well established in the finance literature (see, e.g., Longstaff and Schwartz, 1995; Leland and Toft, 1996).

We decide to remain on an abstract level and model $\tilde{r}_A$ based on aggregate asset classes. If deemed necessary, it is straightforward to adopt a more detailed categorization. Thus, the total asset return consists of the individual returns for each asset class in the portfolio of the life insurer:

$$\tilde{r}_A = \sum_{i=1}^{n} w_i \tilde{r}_i, \tag{19}$$

with

- w_i: portfolio weight for asset class i,
- $\tilde{r}_i$: return of asset class i between $t = 0$ and $t = 1$,
- n: number of asset classes in the portfolio.

In general, the value of the life insurance liabilities in $t = 0$ equals the discounted expected future payments to those insured. For each policy, the future cash flow streams need to be estimated based on actuarial assumptions, taking into account the insured's age and mortality profile, the obligations arising under the contract, as well as applicable embedded options such as interest rate guarantees. While it is common to employ an actuarial technical discount rate, in a solvency measurement context we are interested in the market value rather than the actuarial value of the liabilities. Hence, the current term structure should be used for discounting, and the resulting market value of the life insurance liabilities is not only sensitive to the life expectancies of those insured, but also reacts to changes in the prevailing interest rate environment. If the market value of the liabilities is assumed to continuously increase (or decrease) throughout the period at a stochastic rate, we obtain the following relationship:

$$\tilde{L}_1 = L_0 \exp(\tilde{g}_{\mathrm{L}}), \tag{20}$$

where

- $\tilde{L}_1$: stochastic market value of the liabilities in $t = 1$,
- L_0: deterministic market value of the liabilities in $t = 0$,
- $\tilde{g}_{\mathrm{L}}$: stochastic growth rate of the liabilities between $t = 0$ and $t = 1$.

Similar to the asset side, the aggregated growth rate of the liabilities is determined by individual growth rates for the different categories of liabilities in the life insurance portfolio:

$$\tilde{g}_L = \sum_{i=1}^{l} v_i \tilde{g}_i, \tag{21}$$

with

- v_i: portfolio weight for liability category i,
- $\tilde{g}_i$: growth rate of the liability category i between $t = 0$ and $t = 1$,
- l: number of liability categories in the portfolio.

Furthermore, in order for the company's assets and liabilities to be correlated, let the random variables $\tilde{r}_A$ and $\tilde{g}_L$ adhere to a joint cdf $F(x, y)$:

$$F(x, y) = P(\tilde{r}_A \leq x, \tilde{g}_L \leq y). \tag{22}$$

At this point, we deliberately refrain from making specific distributional assumptions for the asset class returns. Instead, we aim to determine an adequate choice on the basis of distribution fitting in Section 5. Similarly, in contrast to the SST standard model, we do not set the means to zero but explicitly stress the flexibility of describing the return distributions as precisely as possible, including mean, standard deviation, and potentially higher moments.

Based on the stochastic assets and liabilities, it is now possible to derive a distribution for $R\tilde{B}C_1$ and, in turn, the change in risk-bearing capital between $t = 0$ and $t = 1$, which we define according to Equation (12). Analogously to the SST standard approach, this internal model can be extended by scenarios to gain additional flexibility and to capture the tails of the distribution in a more realistic way. Once the aggregate cdf for $\Delta R\tilde{B}C$ has been estimated, the capital requirements can be calculated based on the applicable risk measure, i.e. the $\text{VaR}_{0.5\%}$ (European Union) or the $\text{TVaR}_{1\%}$ (Switzerland).

Table 3 summarizes the main characteristics of the solvency models introduced in this section.

Model characteristic	Solvency II	SST	Internal Model
Type	Static stress factor model	Dynamic risk factor model	Structural credit model
Risk measure	value at risk confidence level: 99.5%	tail value at risk confidence level: 99%	Depends on prevailing regulatory regime
Set-up	Solvency capital requirement (SCR) determined by the impact of preset shocks on the net asset value (NAV)	Calculation of target capital (TC) based on the distribution of the change in risk bearing capital (ΔRBC), derived by means of 79 risk factors	Direct modeling of stochastic asset and liability market values
Distributional assumptions	Properties of empirical distributions enter preset stress factor values	Multivariate normally distributed risk factors with N(0,Σ)	Flexible distributional assumptions, including empirical means, standard deviations, and correlation structure
Calibration	Fully prescribed by CEIOPS based on analysis of time series data	Mainly set by FINMA based on time series data for the latest 10-year period; own calibration for some risk factors possible (e.g., private equity)	Generally flexible; selection of index return time series data and estimation period needs to be accepted by the regulator
Scenarios	n/a	Preset capital market scenarios capture the tail characteristics of ΔRBC	Flexible

Table 3: Comparison of Characteristics of the Three Market Risk Models

5 Implementation of the Market Risk Models

5.1 Model Calibration

5.1.1 Stylized Balance Sheet of a Representative Life Insurer

Prior to an implementation of the previously explained solvency models based on real-world data, we need to set out the characteristics of the representative life insurance company whose capital requirements we would like to determine. Table 4 shows the stylized balance sheet underlying our calculations. To enhance the comparability of our results, we abstract from the distinction between Euros (Solvency II) and Swiss Francs (SST) by denominating the balance sheet in currency units (CU). The total asset value and portfolio weights are based on the 2009 financial statements for 21 Swiss life insurers, which are available from the FINMA insurance report portal (see FINMA, 2009). We have averaged the respective figures over all companies and subsequently aggregated some of the items to stylize the balance sheet. Although annual reports are not consistent with market values as required by the solvency frameworks under consideration, we deem this be the most adequate and reliable proceeding in the absence of market value balance sheets. The firm's equity ratio is also based on figures from the insurance report portal, which range from 5 percent to 12 percent. In general, the difference between market and book values is greater on the asset than on the liability side, implying that a market value balance sheet should exhibit more equity capital. Thus, we decided to employ the upper bound of 12 percent for the equity ratio. Furthermore, investment limits have been retrieved from the applicable regulatory directives (see FOPI, 2008). The maximum percentage of the total assets that insurers are allowed to allocate to the category of alternative investments, for example, is 10 percent.[14] As we do not have any details on the constituent positions within the company's asset categories, we assume that the structure of each subportfolio equals that of a common capital market index. The life insurer's U.S. government bond portfolio, for example, is represented by the S&P U.S. Treasury Index. Modified durations for all three bond portfolios (U.S., EU, and Swiss government bonds) as of December 31, 2010, have been obtained from Bloomberg. The aggregate asset duration equals the weighted average of the bond portfolio durations and amounts to 4.10.[15] Finally, we set the duration of the company's life insurance liabilities to 10.00, implying a duration gap of 5.90. These values are in line with estimations of various practitioner studies for the German life insurance industry (see Finke, 2006, Steinmann, 2006, and Linowski, 2007).

5.1.2 Solvency II Market Risk Module

Our calibration of the Solvency II standard formula is consistent with the latest CEIOPS directives (see CEIOPS, 2010a; CEIOPS, 2010b; CEIOPS, 2010c), which take the experience from the global financial crisis into account. Table 5 shows the parameter values for the equity, interest rate, and property risk

[14] In addition, no single fund in this category must amount to more than one percent of the insurer's total assets. Similarly, for fund of funds this proportion is 5 percent. The term "alternative investments" comprises the asset classes of private equity, hedge funds, currency overlays, commodities, as well as structured products (see FOPI, 2008).

[15] By employing a single duration figure for the whole asset side, we implicitly assume that interest rates in the USD, EUR, and CHF area move in lockstep. Since, for our calibration horizon, all pairwise correlations of the changes in the respective average interest rates exceed 0.70, we believe that this is an acceptable simplification.

Assets	Index Representing Portfolio	Value (in CU million)	% of Total Assets	Investment Limit (see FOPI, 2008)	Duration (as of 12/31/2010)
Stocks	–	1,690	13%	30%	–
U.S. Stocks	MSCI USA	390	3%	–	–
European Stocks	MSCI EU	650	5%	–	–
Swiss Stocks	MSCI CH	650	5%	–	–
Bonds		8,580	66%	n.a.	–
U.S. Government Bonds	S&P U.S. Treasury Index	1,430	11%	–	4.23
EU Government Bonds	S&P EU Government Bond Index	1,430	22%	–	6.04
Swiss Government Bonds	Swiss Government Bond Index	2,860	33%	–	7.00
Real Estate	Rued Blass Real Estate Index	1,300	10%	25%	–
Alternative Investments		260	2%	10%	–
Hedge Funds	HFRX Global Hedge Fund Index	130	1%		–
Private Equity	LPX50/PEPI/CAPEI	130	1%		
Cash	Swiss Money Market	1,170	9%	n.a.	–
Total		13,000	100%		4.10

Equity and Liabilities	Value (in CU million)	% of Total Assets	Duration (as of 12/31/2010)
Life Insurance Liabilities	11,440	88%	10.00
Equity	1,560	12%	–
Total	13,000	100%	8.80

Table 4: Stylized Balance Sheet of a Representative Life Insurance Company

shocks that enter our calculations. In order to derive the stress factor for the market risk category "global equity", CEIOPS employed historical return time series of the MSCI World Developed Equity price and total return indices. In addition, the tails of the empirical distribution were taken into account through extreme value theory. These considerations resulted in a 45 percent stress factor for "global equity", which is supported by a majority of the EU member states (see CEIOPS, 2010c). Furthermore, the preset stress factor for the category "other equities" is based on benchmark indices for the four distinct asset classes private equity, hedge funds, commodities, and emerging market equities. In the case of private equity, for example, CEIOPS relied on the return distribution of the LPX50, from which they derived a $\mathrm{VaR}_{0.5\%}$ of 68.67 percent. In contrast to that, their analysis of the hedge fund index HFRX suggested a stress factor of 23.11 percent. As indicated by this discrepancy, the four subcategories drawn together under "other equities" exhibit a considerable heterogeneity. Hence, it would be highly appropriate to introduce a separate stress factor for each of these four asset classes. Nevertheless, CEIOPS insists on a common stress factor of 55 percent (see CEIOPS, 2010c). For global and other equities they proposed a correlation coefficient of 75 percent.[16]

With regard to interest rate risk, CEIOPS provides an upward and a downward shock for each maturity of the term structure.[17] In order to simplify the analysis, we assume a single interest rate for each of the three currency zones (USD, EUR, and CHF) covered by the insurer's bond portfolios. These flat term structures are calculated by averaging the constituent rates of the respective yield curves on December 31, 2010. In line with this proceeding, we average the CEIOPS stress factors for all maturities. Consequently, we get a single upward and downward interest rate shock of +42 percent and -39 percent, respectively. The firm's bond portfolios and life insurance liabilities are assumed to react to these shocks according to their durations.

Finally, CEIOPS based the calibration of the property risk stress factor on the Investment Property Databank, which provides comprehensive historical total return index data for the U.K. They recommend an overall stress factor for property risk of 25 percent, since the descriptive statistics and the lower percentiles of the empirical return distributions were found to be relatively homogeneous across different property classes (see CEIOPS, 2010b).

5.1.3 SST Standard Model for Market Risk

The market risk standard approach of the 2011 SST comprises a total of 79 risk factors, which have been calibrated by FINMA based on time series data between January 2001 and December 2010.[18] To facilitate our analysis and enhance comparability with the internal model, we decided to reduce the number of risk factors in line with the balance sheet structure of our representative life insurance company. Consequently, from the range preset by the regulator, one appropriate risk factor is adopted for each asset subportfolio. All risk factors underlying our calculations together with the estimated volatilities of their

[16] The CEIOPS advice for the standard formula also includes a so-called "symmetric adjustment mechanism", which allows to change the equity stress factors in times of financial crises (see CEIOPS, 2010a). We abstract from this feature.
[17] The respective figures can be found in the Solvency II calibration paper (see CEIOPS, 2010c).
[18] For a complete list of the risk factors and their parameter estimates, refer to the 2011 SST template on the FINMA website.

changes (σ_i) are summarized in the central section of Table 5. Since the life insurer's stock portfolios are assumed to resemble the MSCI country indices (MSCI USA, MSCI EU, MSCI CH), those are employed as equity risk factors. With respect to interest rate risk, we again use flat term structures. Fluctuations in the USD, EUR, and CHF interest rates are translated into value changes for the company's bond portfolios and life insurance liabilities by means of the modified durations in Table 4. In addition, the Rued Blass Index (RBREI) as well as the HFRX serve as risk factors for the real estate and hedge fund portfolio, respectively.[19] To prevent an understatement of risk due to index measurement issues, FINMA requires a doubling of the volatility for the hedge fund risk factor (see FINMA, 2010b).

Insurers are generally allowed to select an own risk factor for private equity. However, if the company is incapable of assigning proper volatility and correlation estimates to the asset class, a standard deviation of 37.50 percent needs to be applied and the correlations to all other risk factors are set to one. To avoid this strikingly detrimental calibration prescribed by FINMA, one of the private equity indices introduced in Section 3.3 could be adopted as risk factor. Their suitability for this purpose, however, needs to be assessed on a case-by-case basis. The degree of diversification of an insurer's private equity portfolio as well as the credibility it assigns to valuations provided by its general partners are key aspects to be taken into account for the selection. To get a more complete picture, we will consider the LPX50, the PEPI, and the CAPEI as calibration alternatives. Just as for hedge funds, FINMA doubts the adequacy of risk figures estimated from private equity indices (see FINMA, 2010b). Hence, the volatilities of the appraisal-based PEPI and CAPEI have also been doubled for the analysis.

As explained in Section 4.2, the means of all risk factor changes are set to zero. $\Delta\tilde{f}_i$ represents absolute deviations for interest rates and log-returns for the other risk factors. The corresponding correlation matrix can be found in Table 9 of Appendix B. In order to further simplify the analysis, we have set the market value margin MVM to zero, which implies $L_t = L_t^b$ as well as $RBC_t = A_t - L_t$.[20] Finally, we complement the analytical part of the SST market risk model with the eleven capital market stress scenarios provided by FINMA. The occurrence probabilities (p_j) and risk factor shocks that govern the total change in risk-bearing capital (c_j) for each scenario can be found in Table 8 of Appendix B.

5.1.4 Internal Model for Market Risk

To ensure a high level of comparability, we want to align the calibration of our asset-class-based internal model as closely as possible with the risk-factor-based SST standard approach.[21] Therefore, where possible, the indices that represent the life insurer's asset portfolios (see Section 5.1.1) have been chosen so as to correspond to a market risk factor of the SST. Exceptions are the firm's U.S., European, and

[19] Note that we deliberately choose the HFRX over the HFRI, since it comprises around 40 instead of more than 2,000 funds and its constituents must be open for new investment (see www.hedgefundresearch.com). Swiss insurers are allowed to allocate up to 10 percent of their portfolio to alternative investments. Yet, any single fund must not account for more than one percent of the total assets (see investment limits in Table 4). This implies that portfolios with as little as 10 funds are still admissible. Thus, we believe that the HFRX is more suitable to represent the typical hedge fund portfolio of a Swiss insurance company.

[20] Refer to Equations (9) and (10).

[21] Due to its simplistic design, a reasonable alignment with the Solvency II standard formula hardly seems feasible.

Solvency II (Shocks)	Down %	Up %	**SST** (Risk Factors)	$\mathrm{E}\left[\Delta\tilde{f}_i\right]$ % p.a.	σ_i % p.a.	**Internal Model** (Asset Classes)	$\mathrm{E}\left[\tilde{r}_i\right]$ % p.a.	$\sigma_{\tilde{r}_i}$ % p.a.
Global Equities			**Equity markets**			**Stocks**		
U.S. Stocks	–45.00	–	MSCI USA	0.00	16.73	MSCI USA	1.48	16.73
European Stocks	–45.00	–	MSCI EU	0.00	20.29	MSCI EU**	0.00	20.29
Swiss Stocks	–45.00	–	MSCI CH	0.00	15.24	MSCI CH	0.18	15.24
Interest Rate Risk			**Interest Rates**			**Bonds**		
USD Interest Rate	–39.00	+42.00	USD Interest Rate	0.00	0.95	S&P USTI	4.91	4.29
EUR Interest Rate	–39.00	+42.00	EUR Interest Rate	0.00	0.68	S&P EUGI	4.76	3.63
CHF Interest Rate	–39.00	+42.00	CHF Interest Rate	0.00	0.54	SIX SBI	4.20	3.83
Property Risk			**Real Estate**			**Real Estate**		
Swiss Real Estate	–25.00	–	RBREI	0.00	6.51	RBREI	6.72	6.51
Other Equities			**Hedge Funds**			**Hedge Funds**		
Hedge Funds	–55.00	–	HFRX*	0.00	5.96	HFRX*	3.50	5.96
			Private Equity			**Private Equity**		
Private Equity	–55.00	–	1) FINMA	0.00	37.50			
			2) LPX50	0.00	26.70	1) LPX50**	0.00	26.70
			3) PEPI*	0.00	11.73	2) PEPI*	6.97	11.73
			4) CAPEI*	0.00	11.61	3) CAPEI*	9.60	11.61
						Cash		
						SMMI	1.21	0.31

Table 5: Input Data for the Three Market Risk Models

Solvency II shocks as well as means and standard deviations for the SST risk factors and the asset class return distributions of the internal model. The latter two are based on the period 01/01/2001-12/31/2010. Indices: MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); Rued Blass Real Estate Index (RBREI); HFRX Global Hedge Fund Index (HFRX); LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI); Swiss three-month Money Market Index (SMMI).

Notes:
* As required by FINMA, the volatilities for the HFRX and the appraisal-based private equity indices are doubled for the SST.
** Negative means have been set to zero.

P-Values	**LPX50**	**PEPI**	**CAPEI**	**MSCI USA**	**MSCI EU**	**MSCI CH**
K-S Test	0.0430	0.2453	0.5716	0.1033	0.2225	0.1281
A-D Test	0.0276	0.3510	0.5760	0.1593	0.1067	0.1527
P-Values	**S&P USTI**	**S&P EUGI**	**SIX SBI**	**HFRX**	**RBREI**	**SMMI**
K-S Test	0.5946	0.2660	0.9630	0.0122	0.3659	0.0004
A-D Test	0.6249	0.4013	0.9651	0.0156	0.5585	0.0023

Table 6: Goodness-of-Fit Test for the Index Return Time Series (01/01/2001 - 31/12/2010)

P-values for the Kolmogorov-Smirnov (K-S) and the Anderson-Darling (A-D) test of the null hypothesis that the sample has been drawn from a normal distribution. Indices: LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI); MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); HFRX Global Hedge Fund Index (HFRX); Rued Blass Real Estate Index (RBREI); Swiss three-month Money Market Index (SMMI).

Swiss government bond holdings, which the internal model captures directly through the historical return time series of the S&P USTI, the S&P EUGI, and the SIX SBI instead of resorting to the underlying interest rates as risk factors.[22] Nevertheless, consistency is ensured, since our calculations for both the Solvency II and SST standard approach are based on the modified durations of these bond index portfolios in combination with the respective interest rate shocks or volatilities. Again, the LPX50, the PEPI, and the CAPEI are utilized as alternative proxies for the private equity portfolio. In addition, we exploit the flexibility of the internal model to explicitly account for the life insurer's cash holdings through the Swiss 3-month Money Market Index (SMMI).

In Section 4.3, we deliberately left the probability distributions for the asset class returns undefined. By means of the Kolmogorov-Smirnov (K-S) and the Anderson-Darling (A-D) goodness-of-fit statistic, we now test whether the normal distribution adequately describes the observed returns of our index portfolios. In line with the calibration horizon of the SST market risk standard approach, we employ the return time series for each index from January 2001 to 2010. From the resulting p-values reported in Table 6 only those for the LPX50, the HFRX, and the SMMI are smaller than 0.05. For the other 12 time series, in contrast, we cannot reject the null hypothesis of normality on the five percent significance level. Therefore, to facilitate the analysis and further enhance comparability with the SST standard model, we deem it acceptable to assume normally distributed asset returns. Consequently, $\tilde{A}_1$ is log-normally distributed. Mean $\mu_{\tilde{r}_i} = \mathrm{E}[\tilde{r}_i]$ and standard deviation $\sigma_{\tilde{r}_i}$ for the empirical return distribution of each asset class are shown in the right section of Table 5 and the respective correlation matrix can be found in Table 7 of Appendix B. This μ–σ–approach for the asset model is well grounded in the classical portfolio theory (see Markowitz, 1952). Note that the $\mathrm{E}[\tilde{r}_i]$ of the MSCI EU as well as the LPX50 have been set to zero, since we obtained negative estimates for the chosen calibration period.

Due to the lack of publicly available time series data, reflecting the behavior of the technical reserves of life insurers, we face a challenge with regard to the calibration of the stochastic liabilities in our internal model. In order to overcome this issue we assume normally distributed liability growth rates, abandon the detailed categorization of the liability side as implied by Equation (21), and resort to approximations on an aggregate level. Since life insurance liabilities are commonly valued using actuarial methodology, we decide to draw on the current maximum technical interest rate i_{tec} in Switzerland which is published on the FINMA website as proxy for the mean growth rate of the liabilities:

$$\mathrm{E}[\tilde{g}_L] = i_{\text{tec}} = 0.0175. \tag{23}$$

Furthermore, suppose that the life insurance liabilities exclusively react to fluctuations in the CHF interest rate. Based on this assumption, we estimate the volatility of their growth rate $\sigma_{\tilde{g}_L}$ as follows:

$$\sigma_{\tilde{g}_L} \approx \sigma_{i_{\text{CHF}}} \cdot D_L = 0.0054 \cdot 10.00 = 0.05, \tag{24}$$

[22] A more sophisticated approach could aim at deriving a return distribution for the bond portfolios based on a stochastic interest rate model such as, e.g., the ones proposed by Vasicek (1977) or Cox et al. (1985). However, in the absence of detailed information (notional, maturities, coupons, etc.) on the bond portfolio constituents, we opt for this alternative.

where $\sigma_{i_{\text{CHF}}}$ is the volatility of the CHF interest rate (see Table 5) and D_L stands for the modified duration of the life insurance liabilities (see Table 4). Intuitively, this means that we expect the life insurance liabilities to be roughly 10 times as volatile as the underlying CHF interest rate.[23] Owing to these considerations, $\tilde{L}_1$ is log-normally distributed as well.

Having determined the marginal distributions of $\tilde{A}_1$ and $\tilde{L}_1$, we still need to introduce a dependency structure for these two random variables as provided for by Equation (22). In the absence of empirical evidence for a nonlinear relationship, we opt for a linear correlation of $\tilde{r}_A$ and $\tilde{g}_L$ governed by the following bivariate normal distribution:

$$(\tilde{r}_A, \tilde{g}_L)' \sim N_2(\mu, \Sigma_{\tilde{r}_A, \tilde{g}_L}), \tag{25}$$

with the two-dimensional mean vector μ and the variance-covariance matrix $\Sigma_{\tilde{r}_A, \tilde{g}_L}$. To estimate the corresponding correlation $\rho_{\tilde{r}_A, \tilde{g}_L}$, we employ the following approximation:

$$\rho_{\tilde{r}_A, \tilde{g}_L} \approx \begin{cases} D_A/D_L & \text{if} \quad D_A \leq D_L \\ D_L/D_A & \text{otherwise} \end{cases}, \tag{26}$$

where D_A represents the modified duration of the asset side (see Table 4). In our context, D_A and D_L are assumed to be strictly positive. The intuition behind this approach is that the joint variation of assets and life insurance liabilities should arise because they are both sensitive to interest rate fluctuations.[24] Thus, the higher the duration gap between assets and liabilities, the lower their correlation coefficient in the context of our internal model. Inserting the respective figures, we obtain

$$\rho_{\tilde{r}_A, \tilde{g}_L} \approx \frac{D_A}{D_L} = \frac{4.10}{10.00} = 0.41. \tag{27}$$

Assuming $r_f = 0$ and employing the previously determined distributional characteristics for $\tilde{A}_1$ and $\tilde{L}_1$ in combination with Equation (12), we get $\Delta R\tilde{B}C$ as follows:

$$\Delta R\tilde{B}C = R\tilde{B}C_1 - RBC_0 = (\tilde{A}_1 - \tilde{L}_1) - (A_0 - L_0). \tag{28}$$

Since this is the difference of two log-normal random variables minus a constant, there is no analytical solution for $\text{VaR}_{0.5\%}$ and $\text{TVaR}_{1\%}$. Thus, we will resort to Monte Carlo simulations (with $1,000,000$ iterations) to derive the capital requirements. Finally, for the SST, we shift the ΔRBC distribution of the internal model by the c_j associated with the SST standard scenarios (see Table 8 in Appendix B) and aggregate the resulting distributions according to Equation (17). In addition, we double the volatilites of the HFRX and the two appraisal-based private equity benchmarks as required by FINMA.

[23] Since duration measures are generally based on the assumption of a linear relationship between the value change and the interest rate change, this approximation should hold sufficiently well for small fluctuations. However, it is important to note that, by modeling the liabilities on an aggregate level, we abstract from any diversification effects between different liability categories such as, for example, longevity- and mortality-related risks. Hence, there is a likelihood that we overestimate the volatility to a certain extent.

[24] Again, a corollary of the linear relationship underlying the duration measure is that the accuracy of this approximation deteriorates for larger interest rate movements.

5.2 Market Risk Capital Requirements of the Life Insurance Company

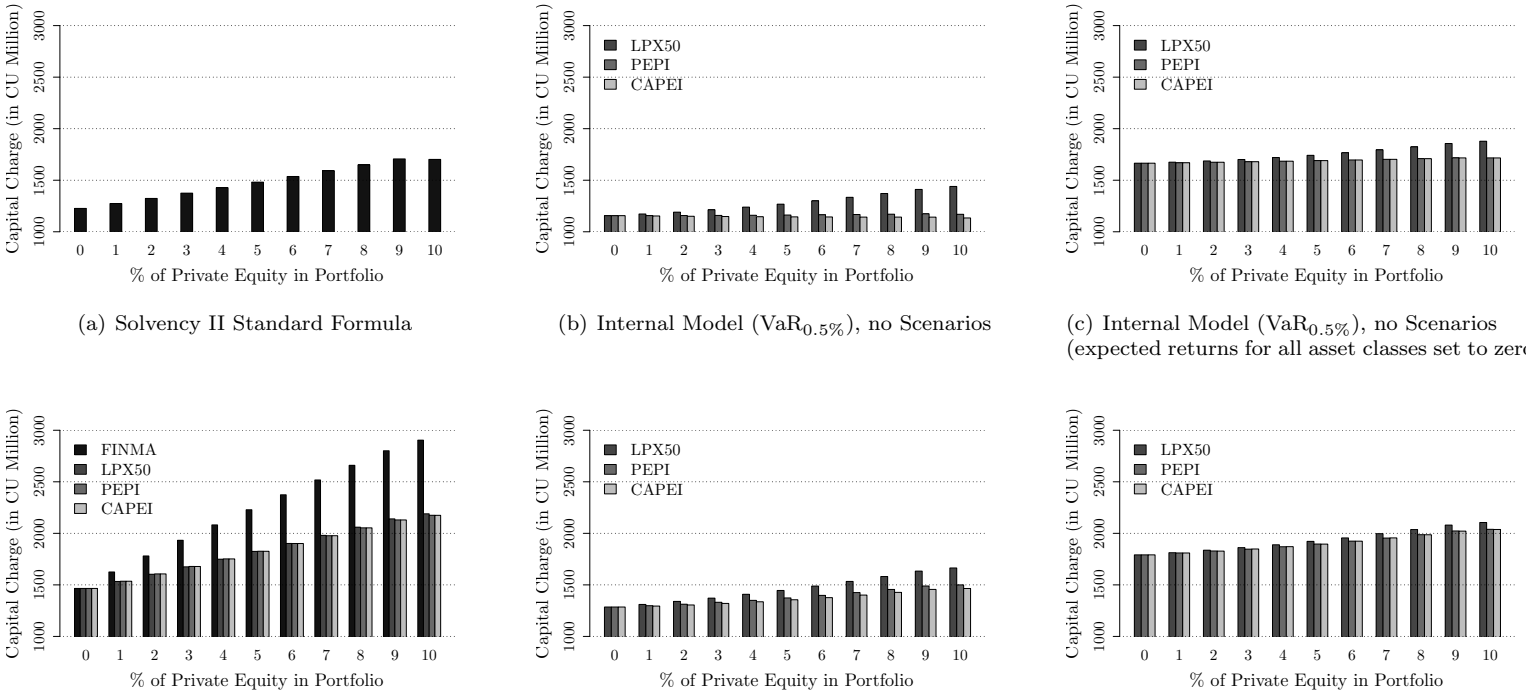
In this section we calculate and compare the market risk capital requirements (in CU) for the representative life insurance company under the Solvency II and the SST standard approaches as well as the internal model. To illustrate the impact of the firm's private equity holdings, we differentiate between the previously discussed calibration alternatives for the asset class and vary the associated portfolio weight between zero and the Swiss legal investment limit of ten percent. Since the weights of all portfolio constituents must sum to 100 percent, the increasing fraction of private equity needs to be funded through a reduction in the other asset classes. In this regard we adopt a procedure suggested by Braun et al. (2011). From the basic asset allocation in Table 4, we calculate the weight of each asset class with regard to the remaining part of the portfolio if private equity is excluded. The resulting percentages will be called "residual weights". Consider the following example: aside from private equity, the remaining asset classes together make up 99 percent of the portfolio, 3 percent of which are U.S. stocks. Consequently, U.S. stocks are assigned a "residual weight" of $3/99 = 3.03$ percent.[25] For each percentage allocated to private equity, the rest of the asset portfolio is then split according to these residual weights. This implies that any increase in the firm's private equity holdings is associated with an absolute decrease in all other asset classes, while their weights relative to each other remain unchanged. It should be noted that each of the varied portfolio structures necessitates an adjustment of the asset duration, which, in turn, alters the correlation between the firm's assets and liabilities. Similarly, the total change in risk bearing capital for each scenario has to be recalculated. Figure 1 illustrates our results.

Solvency II

We begin our discussion with the capital requirements for the Solvency II standard formula in Figure 1(a) and notice that they generally rise with the private equity portfolio weight.[26] Since changes in the private equity holdings affect the risk-return characteristics of the entire asset portfolio as well as the interaction between assets and liabilities, this phenomenon can be attributed to two distinct effects. Firstly, the rise in the capital charges occurs due to a widening of the duration gap between the firm's assets and liabilities, meaning that the risk-bearing capital is less hedged against market risk. As explained above, an expansion of the private equity portfolio reduces the remaining assets, including the firm's bond positions. Consequently, it implies a reallocation of funds from assets with a duration into private equity, which is assumed to be interest rate insensitive. This causes the overall duration of the asset side to decline, while the duration of the life insurance liabilities is unaffected. Secondly, increasing the fraction of private equity in the asset portfolio is equivalent to assigning additional weight to the category "other equities" with its high stress factor of 55 percent at the expense of more favorably treated risk positions.

[25] In the same fashion, we get 5.05 percent for European stocks as well as Swiss stocks, 11.11 percent for U.S. as well as European government bonds, 22.22 percent for Swiss government bonds, 10.10 percent for real estate, 1.01 percent for hedge funds, and 10.10 percent for cash.

[26] Note that we observe a small capital relief when the private equity allocation rises from nine to ten percent. This occurs due to the fact that the legal investment limit of ten percent holds for both hedge funds and private equity. Thus, to be able to invest ten percent of its portfolio in private equity, the insurer needs to completely dissolve its hedge fund holdings. To account for this issue, we decided to reassign the residual weight of hedge funds to Swiss government bonds, which a have much lesser impact on capital requirements.



(a) Solvency II Standard Formula

(b) Internal Model ($VaR_{0.5\%}$), no Scenarios

(c) Internal Model ($VaR_{0.5\%}$), no Scenarios (expected returns for all asset classes set to zero)

(d) SST Standard Approach

(e) Internal Model ($TVaR_{1\%}$), Scenarios

(f) Internal Model ($TVaR_{1\%}$), Scenarios (expected returns for all asset classes set to zero)

Figure 1: Capital Charges for Different Private Equity Portfolio Weights

This figure shows the life insurer's total capital charges for market risk with respect to different proportions of private equity in the asset portfolio: 0% reflects the portfolio without any private equity investments and 10% represents the legal investment limit according to the Swiss Federal Office of Private Insurance (FOPI, 2008). Each subfigure comprises the results for a different model and configuration: (a) Solvency II standard formula; (b) internal model with $VaR_{0.5\%}$; (c) internal model with $VaR_{0.5\%}$ and mean returns for all asset classes set to zero; (d) SST standard approach; (e) internal model with $TVaR_{1\%}$ and SST scenarios; (f) internal model with $TVaR_{1\%}$, SST scenarios, and mean returns for all asset classes set to zero. Alternative calibrations for private equity under the SST standard approach and the internal model: FINMA parameter values; LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).

In Figure 1(b) we have plotted the capital charges that arise when the internal model is run under Solvency II (i.e., based on a $\text{VaR}_{0.5\%}$ and without scenarios).[27] Obviously, for a zero percent weight, the results are independent of the chosen private equity benchmark. Under the LPX50 calibration, more private equity in the portfolio is associated with a clear increase in capital requirements. The previously discussed widening of the duration gap is also a crucial driver here, since a decline in the asset duration leads to a lower correlation between the stochastic assets and liabilities of the life insurer in the internal model (refer to Equation (26)). However, in contrast to the Solvency II standard formula, which exclusively focuses on stress factors, the internal model captures changes in the portfolio structure along two dimensions. Thus, it allows for a second effect that can either counter or reinforce the capital increase through an improvement or deterioration of the overall risk-return-characteristics. This is highlighted in Figure 2, which shows the portfolios from Figure 1 for all three private equity calibration options (LPX50, PEPI, and CAPEI) in the μ–σ– space.

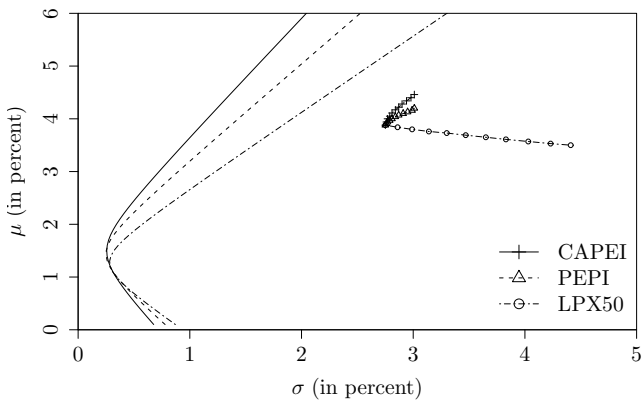


Figure 2: Risk-Return-Profile of the Life Insurer's Asset Portfolio for Different Private Equity Allocations

This figure illustrates the evolution of the life insurer's asset portfolio in the μ–σ–space. The return volatilities σ for each portfolios lie along the x-axis whereas the y-axis represents the corresponding expected returns μ. The crosses, triangles, and circles represent the location of the asset portfolio for different private equity allocations under the three calibration options: LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI). The common point marks the portfolio with 0% private equity. Subsequently, the weight is increased in discrete steps up to 10%. The solid, dashed, and dotdashed lines indicate the corresponding efficient frontiers.

The common point reflects the portfolio without any private equity (zero percent weight in Figure 1). Starting from there, if the asset class is assumed to be represented by the LPX50, the rising proportion of private equity effects a shift further away from the efficient frontier, thereby intensifying the rise in capital requirements. The reason being that, due to the low expected return (0 percent p.a.) and the high standard deviation (26.50 percent p.a.) of the LPX50, the rebalancing towards private equity reduces μ and increases σ of the overall asset portfolio. For the PEPI and the CAPEI calibrations with their attractive performance characteristics, in contrast, the additional private equity exposure moves the

[27] Recall that our estimation period from 2001 to 2010 has been chosen so as to match the requirements of the 2011 SST. Yet, as an additional robustness check we have split this period in two halves: 2001 to 2005 (i.e., excluding the subprime financial crisis) and 2006 to 2010 (i.e., mainly comprising the financial crisis). The corresponding results for the internal model are reported in Appendix C. Although the observed effects are of a somewhat different magnitude, their direction is stable.

portfolio alongside the efficient frontier to the upper right in the μ–σ–space. As can be observed in Figure 1(b), this notably mitigates the increase of the capital charges under the PEPI relative to the LPX50 calibration and the standard approach. Furthermore, with the CAPEI calibration we even document a slight reduction in capital charges of 22 mn between the zero and ten percent private equity weight, which indicates that the impact of the enlarged duration gap is surmounted by the transformation of the risk-return profile on the asset side.[28] The differences in the firm's capital requirements for the three private equity proxies become larger when the proportion of private equity in the portfolio increases. At this point it is important to emphasize that the capital charges associated with the PEPI and the CAPEI should be considered as a theoretical lower bound, since these indices are not based on market values and could thus be distorted (refer to the issues raised in Section 2). With its 50 constituents, the LPX50 is a lot less diversified than the PEPI and the CAPEI, which comprise several hundred limited partnership funds. Apart from that, it has performed poorly during the relevant time period (see Section 3.3). Therefore, we view the capital requirements arising from a calibration with the LPX50 as an upper bound.[29]

Finally, comparing Figures 1(a) and (b) we find that, for any given private equity portfolio weight, the insurer faces a lower solvency capital requirement when employing an internal model. This holds true for each considered calibration option and can be attributed to the fact that the latter takes the investments' expected returns into account. The magnitude of this effect can be assessed through Figure 1(c), which shows the capital charges under the internal model when the expected returns for all asset classes are set to zero. Due to their major impact on the results, neglecting statistical means can evidently cause severe biases in the results. Against this background it should be welcomed that, in the context of modern solvency frameworks, regulators aim to encourage insurers to develop own internal models, which best fit their risk situation and risk management processes (see, e.g., EC, 2010). As they generally provide a higher level of detail and sophistication than the standard approaches, it is possible that these internal models yield lower capital charges. However, for the accreditation of the regulatory authority, insurers will need to demonstrate that the foundations of their models are built upon sound economic reasoning.

Swiss Solvency Test

Figure 1(d) displays our results for the SST standard approach. First of all, similar to our findings for Solvency II, the life insurer's capital charges increase with the percentage of private equity in the portfolio. Once more, two effects work in the same direction. On the one hand, reallocating funds to private equity enlarges the asset-liability duration gap and, on the other hand, it implies a stronger impact of a risk factor with a high volatility. As could be expected, the supervisory parameter values of FINMA turned out to be the most expensive calibration option. To see how inappropriate the 37.50 percent risk factor volatility combined with a full correlation to all other assets actually is, consider the following example. When the private equity holdings are expanded from zero to ten percent, the capital

[28] Unfortunately, this decline in the capital charges is rather difficult to see in Figure 1(b), since the scale of the y-axis has been chosen so as to ensure comparability with the other subfigures.

[29] If the portfolio weight for private equity encompassed both listed and limited partnership private equity, we would generally expect capital charges somewhere in between those for the pure allocation to the LPX50 and the PEPI/CAPEI.

requirements under the standard approach with FINMA parameter values rise by approximately 1.5 bn. Taking into account that the firm's balance sheet total equals 13 bn (refer to Table 4), this is more than the absolute value that corresponds to a private equity weight of ten percent (1.3 bn). In other words, for each currency unit which is redistributed from the remaining, mostly more favorably treated asset classes in the portfolio to private equity, the firm's overall capital charges increase by about 1.15 currency units. Clearly, under these circumstances it is generally not sensible for insurance companies to invest in private equity at all. Furthermore, we see that proxying private equity by the LPX50, the PEPI, or the CAPEI does hardly make a difference under the SST standard approach, since the associated capital charges move virtually in lockstep. This is induced by the fact that the SST standard approach does not account for the means of the risk factor changes, implying that the main driver of the capital charges are the respective standard deviations. The doubled estimates for the returns of the PEPI (23.46 percent p.a.) as well as the CAPEI (23.22 percent p.a.) are very close to the volatility of the LPX50 returns (26.70 percent p.a.). As a result, only small discrepancies in the capital requirements can prevail because the three private equity indices exhibit different correlation structures with the remaining positions in the portfolio. Another phenomenon that is caused by the model set-up of the SST standard approach can be observed in Figure 1(d). Under all four calibration options, the capital charges increase virtually linearly in the private equity portfolio weight. However, the slopes of the corresponding trendlines are not perfectly proportional to the return standard deviations of the indices: the approximately linear slope of the capital charges for the FINMA parameter values equals 3.88 (= 145.43/37.50) times its volatility, which compares to a multiple of 2.79 (= 74.38/26.70), 3.11 (= 72.85/23.46), and 3.14 (= 72.83/23.22) for the LPX50, the PEPI, and the CAPEI, respectively. Again, these slight differences occur due to the respective correlation matrices. Perfect proportionality of the slope to the standard deviation of the risk factor change would require the exact same correlation matrix for all three indices. Intuitively, the FINMA calibration exhibits the highest possible slope for a volatility of 37.50, since, at the same time, all correlations with other asset classes are set to one. If another index with a volatility of 37.50 but lower correlations existed, the slope of its trendline should be flatter.

As shown in Figure 1(e), the increase in capital requirements provoked by an expansion of the firm's private equity investments is less pronounced when the internal model is employed under the SST (i.e., based on a $\mathrm{TVaR}_{1\%}$ and with scenarios). Moreover, the results are now sensitive to the private equity benchmark used for calibration. Both effects arise due to the fact that the internal model captures the full risk-return-characteristics of each asset class. In contrast to Solvency II, however, it is run with doubled volatilities for the appraisal-based indices and the SST scenarios are superimposed on the resulting RBC distribution. Consequently, the movements in the μ–σ–space have a less dampening impact for the PEPI and are no longer strong enough to exceed the effect of the widening duration gap for the CAPEI calibration. Nevertheless, the relatively high mean returns of the PEPI (6.97 percent p.a.) and the CAPEI (9.60 percent p.a.) provide for lower capital charges compared to the LPX50, which entered the analysis with a mean return of zero (see Table 5). The discrepancies between the different calibration options are amplified by a rising private equity proportion in the portfolio.

Finally, independent of the selected calibration alternative, all feasible allocations to private equity

produce lower capital charges under the internal model than under the SST standard approach. Again, this is mainly attributable to the inclusion of expected returns, implying that the internal model establishes a link between the performance characteristics of an asset class and the firm's regulatory capital requirements. Figure 1(f) exhibits the SST market risk capital charges under the internal model in case the means for all return distributions are set to zero. Analogously to our analysis for Solvency II, this illustrates the importance of a μ–σ–approach in the context of solvency measurement.

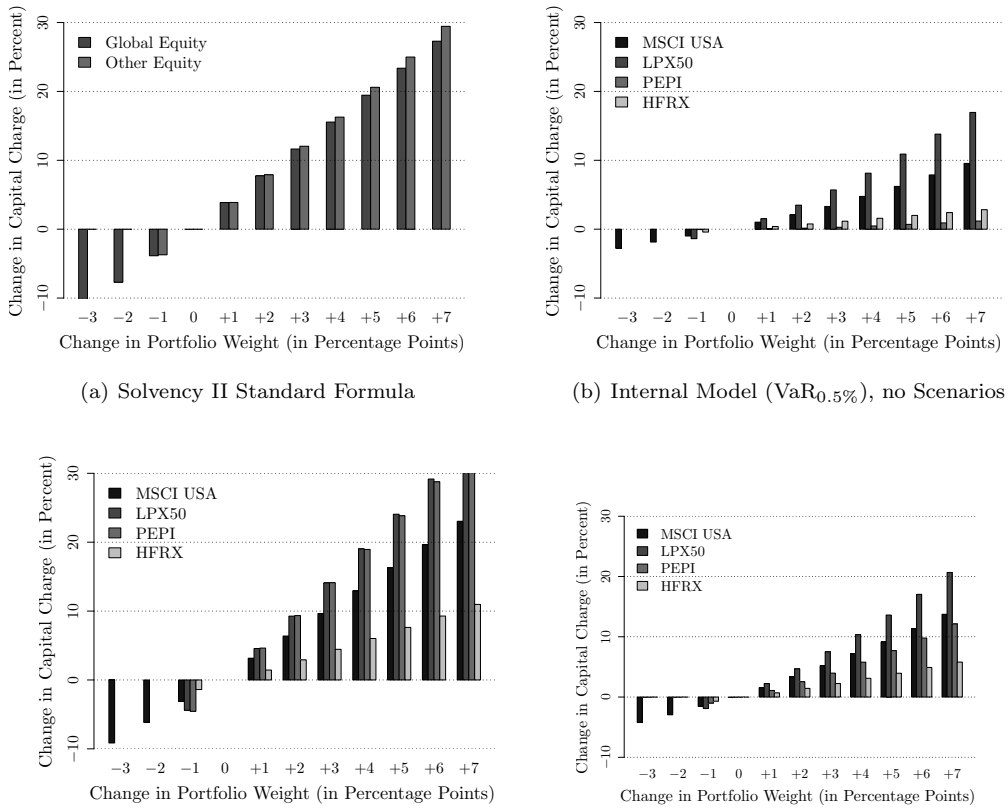
Changes in the Capital Requirements: Private Equity vs. Public Equity and Hedge Funds

Our last analysis is based on a specific decision faced by the life insurer introduced in Section 5.1. Imagine the company plans to allocate further funds to an asset class with a higher return potential than government bonds. This could be aimed at increasing the probability that embedded guarantees of life insurance policies can be met. Based on the results of our performance analysis in Section 3, private equity and hedge funds would be natural candidates for this purpose. Moreover, one would generally also consider the stock market. Apart from the performance characteristics of a prospective investment, however, the life insurer needs to take the associated change in capital requirements into account. Hence, we aim to address the question of how costly it is from a regulatory capital perspective to increase the exposure to private equity in comparison to public equity and hedge funds. Figure 3 illustrates the results of this analysis. On the x-axis we have plotted by how many percentage points the respective portfolio weight is increased or decreased from its base-case value shown in Table 4. Meanwhile, the y-axis represents the associated percentage change in the capital charges.

We begin our discussion with Figure 3(a). As could be expected, under the Solvency II standard formula, adding private equity or hedge fund exposure to the portfolio is more expensive than entering further stock investments, since the former belong to the category "other equities" with its unfavorable stress factor. For the analysis with regard to the internal model, we need to additionally determine which one of the three stock portfolios (U.S., EU, Switzerland) is to be expanded. Due to its slightly better performance characteristics we select the life insurer's U.S. stock portfolio, represented by the MSCI USA. Furthermore, since both appraisal-based private equity indices delivered quite similar results in the previous section, we decided to restrict this analysis to the results for the PEPI and the LPX50. Figure 3(b) shows the outcome for the internal model under Solvency II. In case the LPX50 is used for calibration, increasing the proportion of private equity is more expensive than adding the same number of percentage points to the life insurer's U.S. stock or hedge fund portfolio. In contrast, if the private equity portfolio behaved more like the PEPI rather than listed private equity, its extension would be associated with the least increases in capital charges.

Turning to the results for the SST standard approach in Figure 3(c), we see at first glance that the largest increases in capital charges relate to additional investments in private equity. Again, the LPX50 and the PEPI calibration hardly differ due to the zero mean model assumption as well as the doubled volatility for the PEPI. Expanding the U.S. stock market portfolio (MSCI USA), on the other hand,

is now associated with the second-smallest rises in capital charges. Finally, adding to the HFRX-like hedge fund holdings turns out to be the least expensive option. Figure 3(d) shows the corresponding results when the firm calculates its SST capital charges based on an internal model. Now it is again more attractive to allocate further funds to a private equity portfolio represented by the PEPI than to public equity. To sum up, given the appraisal-based indices proxy the true behavior of the insurer's private equity assets sufficiently well, it can be less costly from a regulatory capital perspective to increase the exposure to private rather than public equity. Under Solvency II, this even holds for hedge funds.



(a) Solvency II Standard Formula

(b) Internal Model ($VaR_{0.5\%}$), no Scenarios

(c) SST Standard Approach

(d) Internal Model ($TVaR_{1\%}$), Scenarios

Figure 3: Percentage Change in Capital Charges for Increasing Portfolio Weights of Risky Asset Classes

Sensitivity analysis of the change in capital charges with respect to increases or decreases of the portfolio weights for private equity, public equity, and hedge funds from their base-case values in Table 4. The x-axis shows by how many percentage points the base case portfolio weight for the considered asset classes is increased or decreased (0 implies no change). The corresponding percentage changes in the capital charges due to the altered portfolio composition are represented by the y-axis. Each subfigure contains the results for a specific model: (a) Solvency II standard formula; (b) internal model with $VaR_{0.5\%}$; (c) SST standard approach; (d) internal model with $TVaR_{1\%}$ and SST scenarios. Indices: MSCI country index for the U.S. (MSCI USA); LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).

6 Economic Implications and Conclusion

In this paper, we conduct an in-depth analysis of the impact of private equity investments on the capital requirements faced by a representative life insurance company under Solvency II as well as the Swiss Solvency Test (SST). Our discussion begins with an empirical performance measurement of the asset class over the period from 2001 to 2010. Subsequently, we review the standard approaches for market risk set out by both regulatory regimes and outline a potential framework for an internal model. Based on an empirical calibration and implementation of these solvency models, it is possible to derive a number of results, which should be of relevance to industry professionals and regulators alike.

Although the chosen examination period is an unfavorable decade for risky asset classes, our empirical performance analysis conveys an overall solid impression of limited partnership private equity funds. Thus, within reasonable investment limits, the asset class should be a fair choice for the purpose of portfolio diversification, especially from the perspective of life insurers with their large bond holdings. Since some uncertainty with regard to the reliability of the employed appraisal-based indices remains, we also believe that relatively large allocations to private equity should only be considered by experienced investors with strong due diligence and manager selection skills. In addition to its performance characteristics, however, the attractiveness of an asset class for insurers also depends on the associated impact on their capital charges. Hence, we turn to the market risk modules of the two most modern solvency frameworks for the insurance industry. By calculating and comparing the capital charges under the Solvency II and the SST standard approach as well as an internal model, we are able to provide evidence that the former disproportionately penalize relatively volatile asset classes such as private equity, which are commonly also associated with higher expected returns. This is mainly attributable to the fact that the Solvency II standard formula relies on a crude stress factor for the category other equities and the SST market risk model solely focuses on volatilities, while setting all risk factor means to zero. Consequently, life insurers aiming to exploit the potential of private equity may expect significantly lower market risk capital requirements when applying an economically sound internal model. This result is shown to be robust against the indices used for calibration as well as the percentage of private equity in the portfolio. Furthermore, we demonstrate that it can even be less costly to increase the exposure to private rather than public equity. Taking these findings into account, the private equity asset class can be an attractive investment alternative for life insurers both from a performance as well as a regulatory capital perspective.

A final note is due with regard to the examined standard approaches. In our opinion, an inappropriate treatment of assets from a solvency perspective has severe economic implications. Life insurers commonly face the challenge of achieving a sufficient return on investments so as to meet the guarantees that are embedded in their underwritten insurance contracts. However, this goal can virtually not be achieved by solely relying on government bonds, especially in low interest rate environments as they are typical for postcrisis periods. Instead, it is focal to enrich the portfolio with asset classes that exhibit a greater return potential. This is exactly where the problems arise. It is a basic principle in modern finance that, barring exceptional investment skills, higher expected returns are only attainable by assuming higher risks. Thus,

investors always need to consider the overall performance of an asset, i.e. the risk-return profile, rather than ignoring one side of the equation. If this fact is not reflected by regulatory frameworks, economic inefficiencies may be the consequence. Inadequate regulatory capital requirements could, for example, lead to an underrepresentation of certain asset classes that suitable to enhance the risk-return profile of a portfolio and allow life insurers to add value through good asset management skills rather than holding large bond portfolios, which, in many cases, could be simply replicated by the customers themselves. Hence, we deem it crucial that solvency models establish a link between the performance characteristics of an investment and the firm's regulatory capital requirements.

Future research could aim at tackling some of the limitations of our results. Firstly, we ignored certain types of market risk within the analysis. Although our study is in itself consistent, it might be interesting to add FX and credit spreads as additional risk drivers. Secondly, being calibrated based on indices instead of more specific portfolio data, all three solvency models are subject to basis risk. Before inferring from our results to specific cases, we therefore recommend to check with great care whether the employed proxies adequately reflect the situation of the respective life insurance company under consideration. If, for example, the private equity portfolio heavily overweighs venture capital investments, one could rerun our analysis based on a suitable venture capital subindex. Thirdly, some private equity specific issues could be addressed in follow-up work. The phenomenon of style drift may become relevant when the life insurance company invests in a rather small number of funds. In this case one could try and explicitly account for such deviations from a fund's stated investment objectives, although they are difficult to model ex-ante. Apart from that, different methodologies that have been suggested in the literature to adjust appraisal-based private equity indices for certain biases and distortions could be considered as well. However, before such an approach can be employed, it needs to be accepted by the respective regulatory authority. Fourthly, along the way we needed to employ a few assumptions and approximations that could be reassessed once new information becomes available. One example is the linear correlation between the assets and liabilities in our internal model. In case future empirical evidence indicates a nonlinear dependency, the correlation coefficient could be substituted by some sort of copula function. Finally, it might be interesting to extend our research question to other types of institutional investors such as banks and pension funds that face different regulatory environments.

Appendix A SST standard approach: $\Delta\tilde{RF}$ and $\Delta R\tilde{B}C$

$\Delta\tilde{RF}$ can either be measured as returns, i.e. relative changes (e.g. for stock prices) or as absolute deviations (e.g. for interest rates). Let $D = \text{diag}\ (\sigma_1, ..., \sigma_k)$ be a diagonal matrix, carrying the observed standard deviations of the k risk factor changes, and let R be the corresponding correlation matrix:

$$R = \begin{pmatrix} 1 & \rho_{1,2} & \cdots & \cdots & \rho_{1k} \\ \rho_{2,1} & 1 & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \rho_{k-1,k} \\ \rho_{k,1} & \cdots & \cdots & \rho_{k,k-1} & 1 \end{pmatrix}, \tag{29}$$

where $\rho_{i,j}$ equals Pearson's correlation coefficient between the change in risk factor i and j. Based on these elements, Σ can be derived as follows: $\Sigma = DRD$. The sensitivities of RBC are its partial derivatives with regard to the risk factors (see FOPI, 2006):

$$\delta_n = \frac{\partial RBC}{\partial f_n} \qquad \forall n \in \{1, ..., k\}\,. \tag{30}$$

Being denominated in CHF per unit of measurement of the risk factor change, these sensitivities represent the increase or decrease in RBC associated with a one-unit change in the respective risk factor. Based on the assumption of a linear relationship between $\Delta\tilde{RF}$ and $\Delta R\tilde{B}C$, the latter can be derived using the vector of sensitivities $s = (\delta_1, ..., \delta_k)'$:

$$\Delta R\tilde{B}C \approx s'\Delta\tilde{RF}. \tag{31}$$

As a consequence, mean and variance of the change in risk-bearing capital can be expressed as follows:

$$E(\Delta R\tilde{B}C) = E(s'\Delta\tilde{RF}) = s'E(\Delta\tilde{RF}) = 0, \tag{32}$$

$$Var(\Delta R\tilde{B}C) = Var(s'\Delta\tilde{RF}) = \mathrm{s}'\Sigma\mathrm{s} = S'RS, \tag{33}$$

where

$$S = \begin{pmatrix} \delta_1 \\ \vdots \\ \delta_k \end{pmatrix} \circ \begin{pmatrix} \sigma_1 \\ \vdots \\ \sigma_k \end{pmatrix} = \begin{pmatrix} \delta_1\sigma_1 \\ \vdots \\ \delta_k\sigma_k \end{pmatrix}, \tag{34}$$

$\sigma = (\sigma_1, \cdots, \sigma_k)'$ is a column vector containing the volatilities of the risk factor changes, and $\circ$ represents the Hadamard product. By inserting Σ in (33), we find $Var(\Delta RBC) = \mathrm{s}'DRD\mathrm{s}$ as an equivalent formulation for the variance.

Appendix B Further Input Data

(a) Downward Stress Scenario

$Corr_u^{Mkt}$	Equity	Interest	Property
Equity	1.00	0.50	0.75
Interest	0.50	1.00	0.50
Property	0.75	0.50	1.00

(b) Upward Stress Scenario

$Corr_u^{Mkt}$	Equity	Interest	Property
Equity	1.00	0.00	0.75
Interest	0.00	1.00	0.00
Property	0.75	0.00	1.00

Table 7: Solvency II Correlation Matrices for the Aggregation of SCR_{Mkt} (see EC, 2010).

Scenario S_j		S_0	S_1	S_2	S_3	S_4	S_5
Probability p_j		98.90%	0.10%	0.10%	0.10%	0.10%	0.10%
Description		Normal Year –	Equity Drop –	Property Crash –	Black Monday (1987)	Nikkei Crash (1990)	ERM Crisis (1992)
U.S. Stocks	%	–	–60.00	0.00	–21.20	–13.80	–1.10
European Stocks	%	–	–60.00	0.00	–38.70	–25.60	–0.40
Swiss Stocks	%	–	–60.00	0.00	–23.20	–26.40	–5.80
USD Interest Rate	bp	–	0.00	0.00	–61.70	128.30	40.30
EUR Interest Rate	bp	–	0.00	0.00	–79.20	158.00	39.30
CHF Interest Rate	bp	–	0.00	0.00	–67.40	109.80	–67.50
Real Estate	%	–	0.00	–50.00	-3.10	–2.20	–1.10
Hedge Funds	%	–	–30.00	0.00	–5.00	–0.80	0.50
Private Equity	%	–	–70.00	0.00	–25.10	–28.70	–4.40

Scenario S_j		S_6	S_7	S_8	S_9	S_{10}	S_{11}
Probability p_j		0.10%	0.10%	0.10%	0.10%	0.10%	0.10%
Description		Mexican Crisis (1994)	LTCM/ Russia (1998)	Dot-Com Bubble (2001)	Global Deflation –	Global Inflation –	Financial crisis (2008)
U.S. Stocks	%	–7.30	–13.90	–32.20	0.00	0.00	–48.40
European Stocks	%	–11.00	–22.50	–42.10	0.00	0.00	–50.30
Swiss Stocks	%	–18.50	–28.40	–35.70	0.00	0.00	–38.80
USD Interest Rate	bp	270.90	–98.00	–123.10	–132.60	136.50	–188.70
EUR Interest Rate	bp	132.30	–57.90	–83.10	–136.50	136.50	–154.30
CHF Interest Rate	bp	151.00	–36.80	–66.30	–127.00	136.50	–109.60
Real Estate	%	–21.50	-3.90	–7.80	0.00	0.00	–10.80
Hedge Funds	%	–3.60	–11.30	–1.90	0.00	0.00	–28.00
Private Equity	%	–11.50	–18.60	–34.10	0.00	0.00	–64.30

Table 8: Risk Factor Shocks Associated with the Scenarios for the SST Standard Approach

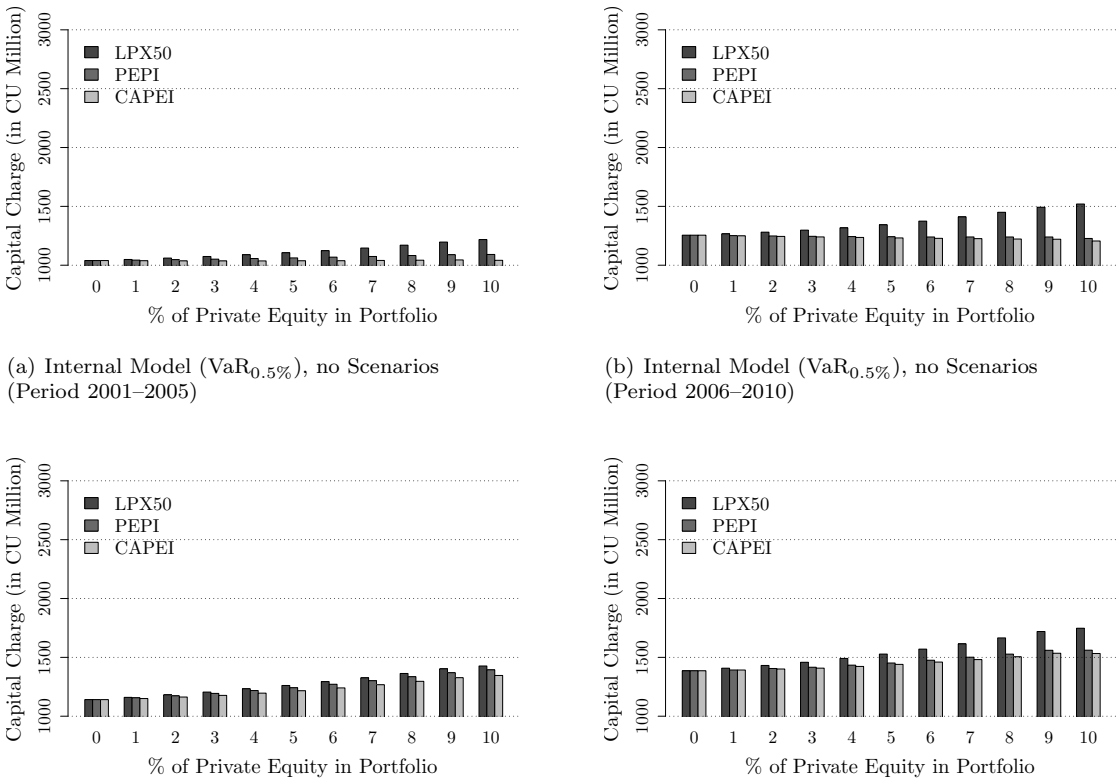
SST Standard	MSCI USA	MSCI EU	MSCI CH	USD Rate	EUR Rate	CHF Rate	RBREI	HFRX	FINMA	LPX50	PEPI	CAPEI
MSCI USA	1.00	0.90	0.80	0.40	0.47	0.36	0.29	0.59	1.00	0.82	0.81	0.80
MSCI EU	0.90	1.00	0.88	0.47	0.49	0.40	0.26	0.54	1.00	0.82	0.78	0.77
MSCI CH	0.80	0.88	1.00	0.48	0.47	0.39	0.13	0.46	1.00	0.79	0.76	0.75
USD Rate	0.40	0.47	0.48	1.00	0.82	0.70	0.10	0.27	1.00	0.42	0.52	0.55
EUR Rate	0.47	0.49	0.47	0.82	1.00	0.83	0.09	0.35	1.00	0.40	0.55	0.57
CHF Rate	0.36	0.40	0.39	0.70	0.83	1.00	–0.03	0.25	1.00	0.32	0.53	0.52
RBREI	0.29	0.26	0.13	0.10	0.09	–0.03	1.00	0.34	1.00	0.29	0.31	0.33
HFRX	0.59	0.54	0.46	0.27	0.35	0.25	0.34	1.00	1.00	0.60	0.63	0.65
FINMA	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00			
LPX50	0.82	0.82	0.79	0.42	0.40	0.32	0.29	0.60		1.00		
PEPI	0.81	0.78	0.76	0.52	0.55	0.53	0.31	0.63			1.00	
CAPEI	0.80	0.77	0.75	0.55	0.57	0.52	0.33	0.65				1.00

Internal Model	MSCI USA	MSCI EU	MSCI CH	S&P USTI	S&P EUGI	SIX SBI	RBREI	HFRX	SMMI	LPX50	PEPI	CAPEI
MSCI USA	1.00	0.90	0.80	–0.34	–0.30	–0.32	0.29	0.59	–0.27	0.82	0.81	0.80
MSCI EU	0.90	1.00	0.88	–0.42	–0.31	–0.35	0.26	0.54	–0.26	0.82	0.78	0.77
MSCI CH	0.80	0.88	1.00	–0.43	–0.30	–0.34	0.13	0.46	–0.28	0.79	0.76	0.75
S&P USTI	–0.34	–0.42	–0.43	1.00	0.73	0.66	–0.08	-0.21	0.13	–0.38	–0.50	–0.52
S&P EUGI	–0.30	–0.31	–0.30	0.73	1.00	0.82	–0.04	–0.20	0.02	–0.26	–0.46	–0.48
SIX SBI	–0.32	–0.35	–0.34	0.66	0.82	1.00	0.04	-0.19	–0.02	–0.27	–0.47	–0.44
RBREI	0.29	0.26	0.13	–0.08	–0.04	0.04	1.00	0.34	–0.16	0.29	0.31	0.33
HFRX	0.59	0.54	0.46	–0.21	–0.20	–0.19	0.34	1.00	–0.19	0.60	0.63	0.65
SMMI	–0.27	–0.26	–0.28	0.13	0.02	–0.02	–0.16	–0.19	1.00	–0.32	–0.43	–0.42
LPX50	0.82	0.82	0.79	–0.38	–0.26	–0.27	0.29	0.60	–0.32	1.00		
PEPI	0.81	0.78	0.76	–0.50	–0.46	–0.47	0.31	0.63	–0.43		1.00	
CAPEI	0.80	0.77	0.75	–0.52	–0.48	–0.44	0.33	0.65	–0.42			1.00

Table 9: Correlation Matrices for the SST Standard Approach and the Internal Model

Indices: MSCI country indices for the U.S. (MSCI USA), Europe (MSCI EU), and Switzerland (MSCI CH); S&P U.S. Treasury Index (S&P USTI); S&P Eurozone Government Bond Index (S&P EUGI); Swiss Government Bond Index (SIX SBI); Rued Blass Real Estate Index (RBREI); HFRX Global Hedge Fund Index (HFRX); Swiss three-month Money Market Index (SMMI); LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).

Appendix C Further Results for the Internal Model



(a) Internal Model ($VaR_{0.5\%}$), no Scenarios (Period 2001–2005)

(b) Internal Model ($VaR_{0.5\%}$), no Scenarios (Period 2006–2010)

(c) Internal Model ($TVaR_{1\%}$), Scenarios (Period 2001–2005)

(d) Internal Model ($TVaR_{1\%}$), Scenarios (Period 2006–2010)

Figure 4: Internal Model: Capital Charges for Alternative Calibration Periods (2001–2005, 2006–2010)

This figure shows the life insurer's total capital charges for market risk under the internal model for the alternative calibration periods 2001–2005 (excluding the financial crisis) and 2006–2010 (mainly comprising the financial crisis). 0% reflects the portfolio without any private equity investments and 10% represents the legal investment limit according to the Swiss Federal Office of Private Insurance (FOPI, 2008). Each subfigure contains the results for a specific model: (a) Solvency II standard formula; (b) internal model with $VaR_{0.5\%}$; (c) SST standard approach; (d) internal model with $TVaR_{1\%}$ and SST scenarios. Alternative calibrations for private equity: LPX50 Listed Private Equity Index; Thomson Reuters Private Equity Performance Index (PEPI); Cambridge Associates U.S. Private Equity Index (CAPEI).

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